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VOLTAGE REGULATED POWER SUPPLIES EMPLOYING TRANSISTORS AND SILICON JUNCTION DIODES

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by

Neri Osborn III

Lieutenant, United States Navy

Submitted in partial fulfillment
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PREFACE

There are a multitude of electronic circuits available for the stabilization of a voltage source. The purpose of this paper is to investigate those utilizing transistors and silicon junction diodes.

The experimental work for this paper was performed at the Phoenix Research Laboratory of Motorola, Incorporated, during the author's industrial tour. Several of the circuits discussed herein were incorporated in equipment designed by Motorola. The others were considered in the light of possible future employment.

The author wishes to gratefully acknowledge the helpful assistance given him by the personnel of Motorola. It is impossible to name all who have contributed their aid. The writer does wish to single out Mr. A. B. Jacobsen who originally suggested the topic. His continued encouragement and critical appraisal of the work while in progress were of inestimable value.

In addition, it is desired to thank Professors G. R. Giet and A. Sheingold for their assistance in preparation for the work performed at Motorola and for their critical appraisal of the final paper.

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TABLE OF SYMBOLS AND ABBREVIATIONS

(Listed in the order of their use in the text)

DC	Direct Current
e_o	Output voltage of regulator
e_i	Input voltage to regulator
i_o	Output Current of regulator
R_L	Load resistance
μ	Variational amplification factor of vacuum tube
g_m	Variational transconductance of vacuum tube
T	Transistor
D	Diode
E_o	DC component of output voltage of regulator
E_i	DC component of input voltage to regulator
$\mathcal{R}$	Voltage regulator regulation factor
r	Voltage regulator output impedance factor
Z_g	Internal impedance of regulator
i_i	Input current to regulator
k	Current loss factor
α	Grounded base current amplification factor
E or e	When used as a subscript refers to emitter
c	When used as a subscript refers to collector
b	When used as a subscript refers to base
β	Grounded emitter current amplification factor
R_D	Dynamic resistance of reference diode
r_e	Equivalent emitter resistance

r_b	Equivalent base resistance
r_m	Equivalent emitter-collector transresistance
r_c	Equivalent collector resistance
I_{co}	DC collector current for zero emitter current
K	Kilohms
μf	Microfarads
KC	Kilocycles
ma	Milliamperes
V_z	Breakdown or reference potential of silicon junction diode
$\mathcal{R}'$	$\frac{de_o}{de_i}$, as defined by equation (3.4)
R_D'	Effective diode dynamic resistance of a shunt regulator
C	Temperature coefficient of reference diode

CHAPTER I

INTRODUCTION

The need for stable DC voltage sources arises in almost every branch of the electronic and electrical field. The voltage output of a rectifier and filter arrangement possesses ripple to a degree dependent on the rectification scheme and the nature and number of filter sections. In addition its constancy is adversely affected by input voltage changes and variations in load. To improve output stability a number of circuit types, generally termed voltage regulators, may be interposed between the unregulated voltage source and the load. See Figure 1.1.

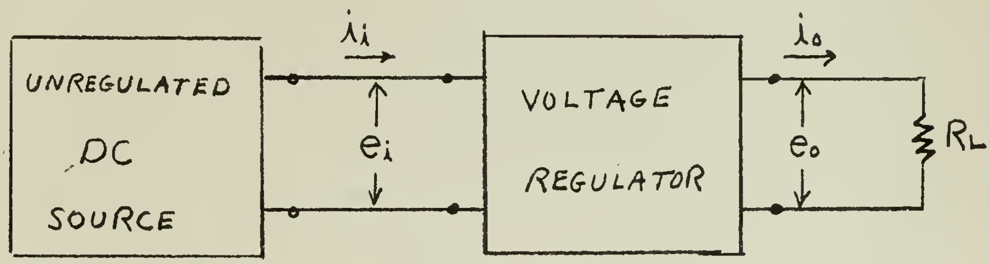

Figure 1.1 Generalized voltage regulator

This paper will concern itself with voltage regulators using transistors and silicon junction diodes as the active and reference elements. Before restricting ourselves it is felt that a discussion of electronic voltage regulators in general is in order.

The desired function of a voltage regulator is the reduction of

variations in output voltage, e_o, with changes in input voltage, e_i and output current i_o. To facilitate the examination of a regulator's action the two functions are considered separately. First we consider the variation of output voltage with input voltage at constant load, or $\frac{\partial e_o}{\partial e_i}$. This is often referred to as regulation. The other item of concern is the change in output voltage with output current at constant input voltage, or $\frac{\partial e_o}{\partial i_o}$. This latter is called the output impedance. These two concepts, regulation and output impedance, describe the characteristics of a voltage regulator. In well designed units both figures can be made quite small.

For background purposes, existing regulator types will be considered. There are numerous excellent articles on the subject of voltage regulators available in the literature. References (1), (2) and (3) are considered to be among the most comprehensive. There has been a great variety of circuit configurations used to accomplish voltage stabilization. The majority of these make use of a cold cathode glow discharge tube, normally referred to as VR-tube. Reference (3) gives a quite complete discussion of this device. Its general characteristics are shown in Figure 1.2.

The simplest voltage regulator is shown in Figure 1.3. It makes use of the non linear impedance characteristics of the VR-tube to maintain the output voltage constant. A complete analysis of this type circuit is presented in a later section.

The VR-tube is also used in conjunction with vacuum tubes in several types of circuits. These regulators use the VR-tube as a constant voltage reference device. Hunt and Hickman[1] classify these

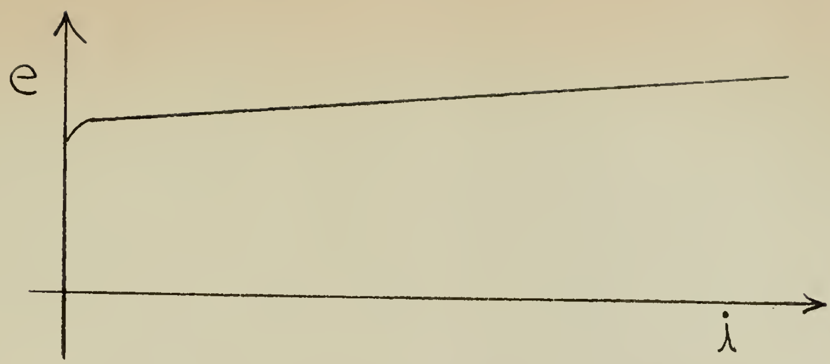

Figure 1.2 Typical VR-tubé Characteristics

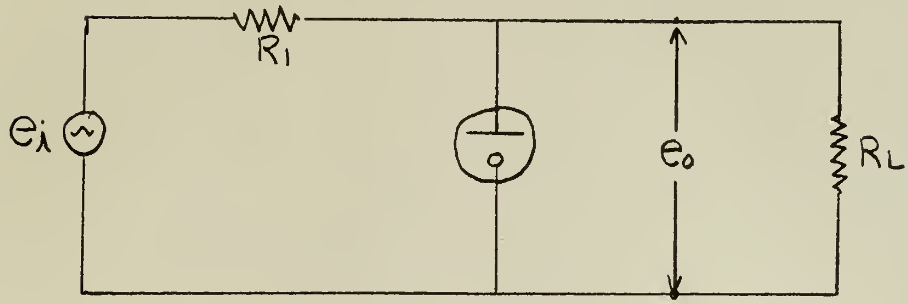

Figure 1.3 Simple VR-tube Voltage Regulator

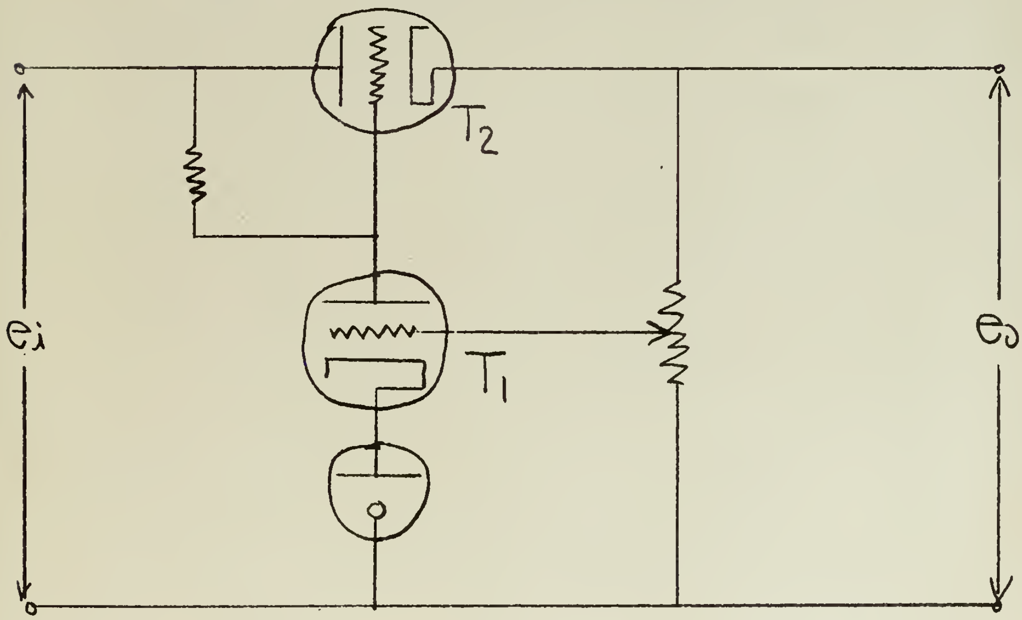

Figure 1.4 Typical Vacuum Tube Series Regulator

in four basic categories: (1) Transconductance type stabilizers (g_m bridge), (2) Amplification factor stabilizers (μ Bridge), (3) Degenerative regulators, (4) Combinations of the first three. Those not familiar with types (1) and (2) above may consult References (1) and (4). The degenerative type is probably the most common. A familiar form is illustrated in Figure 1.4. Analysis of this particular circuit is covered in detail in Reference (2). Reference (3) treats the entire field of degenerative voltage regulators. This class is often referred to as a series regulator because the active element acting as a variable power sink is in series with the load.

In addition to the four categories of Hunt and Hickman, there exist two other vacuum tube regulator circuits that merit separate mention.

The shunt type regulator illustrated in Figure 1.5 is a very common circuit. It uses the vacuum tube to shunt excess current around the load thus keeping the output voltage constant. The circuit of Figure 1.3 can also be considered a shunt regulator.

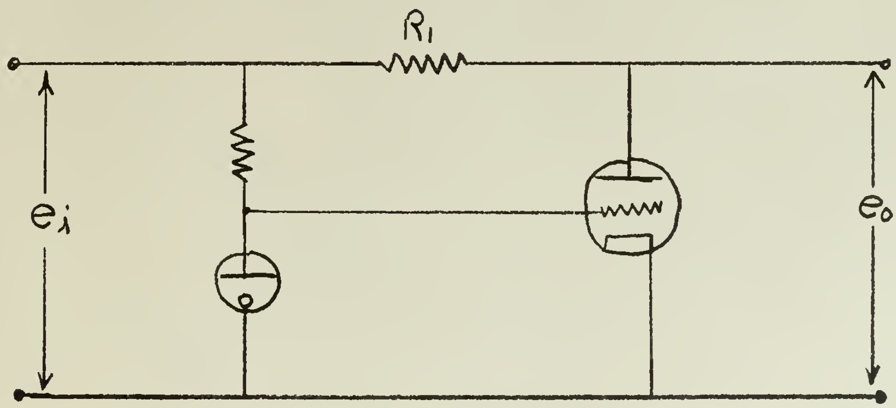

Figure 1.5 Simple vacuum tube shunt voltage regulator

The familiar cathode follower circuit shown in Figure 1.6 is also often used as a voltage regulator. Use is made of its low output impedance to minimize the effects of load current variation. The near unity gain can provide satisfactory regulation if R_1 is made large compared to the dynamic resistance of the VR-tube.

Recent advances in solid state electronics have provided design engineers with a new reference standard. This is the silicon alloy junction diode. When operated in the reverse direction these diodes evince a very low dynamic resistance. Current vs. voltage characteristics for the reverse voltage region of a typical diode are shown in Figure 1.7. They can be seen to be quite similar to those of Figure 1.2 for the VR-tube. Chapter V of this paper and Reference (5) discuss this device in more detail. A salient feature to be noted here is the range of their constant voltage characteristics. These diodes are presently available with reference potentials from about four to over 300 volts. This range is continuous unlike the VR-tube which is commercially available only at 75, 90, 105 and 150 volts.

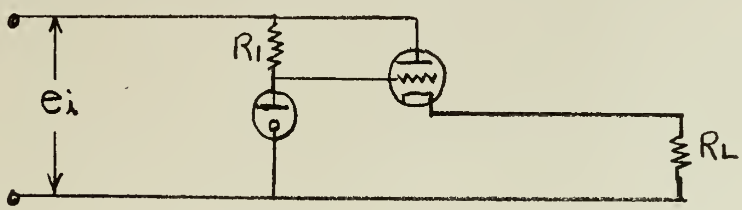

Figure 1.6 Cathode Follower Voltage Regulator

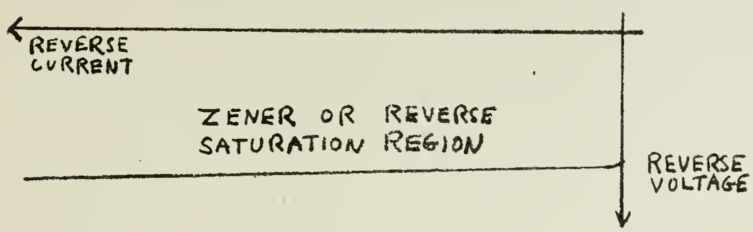

Figure 1.7 Typical Junction Diode Characteristics

The junction diode will probably compete with the VR-tube in their common region for use in conjunction with vacuum tubes. However, it is their low voltage range that appears to be the most attractive at the present. This makes possible a family of voltage regulators employing the transistor as the active element. Prior to the advent of junction diodes, transistors could not readily be used in voltage regulator circuits. This was because the voltages available from VR-tubes were out of the range where transistors operate most effectively.

There are numerous low and medium power constant voltage requirements that can be met by junction diode and transistor voltage regulators. Power supplies for transistor circuits, bias supplies for vacuum tubes, precise laboratory power supplies are a few examples of possible applications. The advantages of these circuits in weight, space and power consumption considerations are readily apparent. Moreover it will be shown in later chapters that the characteristic of the transistor itself prove to be an advantage. Often the transistor analog of an existing vacuum tube circuit proves to be an inherently better voltage regulator.

It is the aim of this paper to examine the characteristics of transistor circuits that can be used for voltage regulation. In order to systematize the fulfillment of this goal the various possible circuit configurations are classified in three categories. Each of the three will be considered in its most simple forms and analyzed theoretically. More involved circuits containing refinements dictated by practical necessity will then be treated in each case. The number of particular circuit configurations that are possible will necessarily make this

paper incomplete in a sense. Nevertheless, the reader should be able to obtain a picture of the worth of these new tools for voltage stabilization. Finally, junction diodes themselves will be considered. Their advantages and limitations as reference elements will be examined.

Lowry[6] has divided transistor voltage regulators into three broad types, shunt, series and emitter follower. His choice appears to be quite logical. Figures 1.8, 1.9 and 1.10 illustrate the three basic regulator circuits. Each possesses its own unique features that makes it useful in various applications.

The transistor shunt type regulator has the same basic function as its counterpart shown in Figure 1.5. It will be demonstrated later that its operation is somewhat different. In essence, one can consider it identical to Figure 1.3, but using a junction diode of improved dynamic impedance.

The series regulator of Figure 1.8 has the same properties as the circuit of Figure 1.4. In the case of both transistors and vacuum tubes the possible configurations for a series, or degenerative, type regulator are numerous. Suffice it to say that each shares the feature of having three basic components: a voltage reference element; a comparison element to generate an error signal proportional to the difference between the output and the reference; a series, or control, element that tends to reduce the error signal towards zero.[3] In other words they act as a servo loop.

The grounded emitter regulator is completely analogous to the cathode follower circuit of Figure 1.6. The fact that the gain of the transistor in the grounded emitter configuration approaches unity more

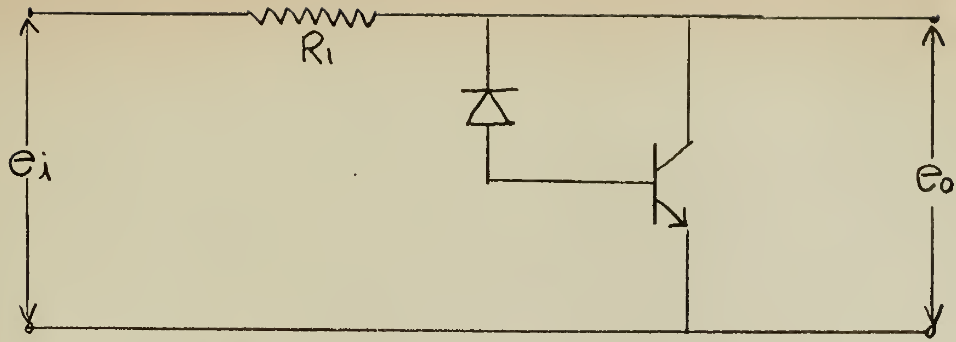

Figure 1.8 Simple Shunt Voltage Regulator

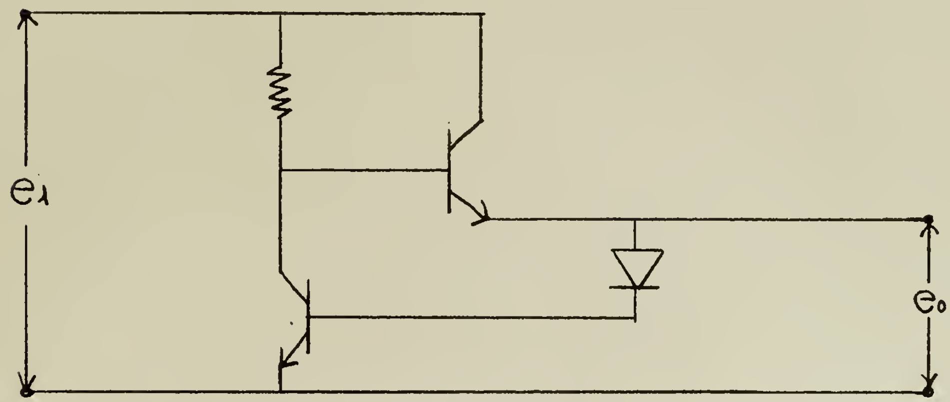

Figure 1.9 Simple Series Voltage Regulator

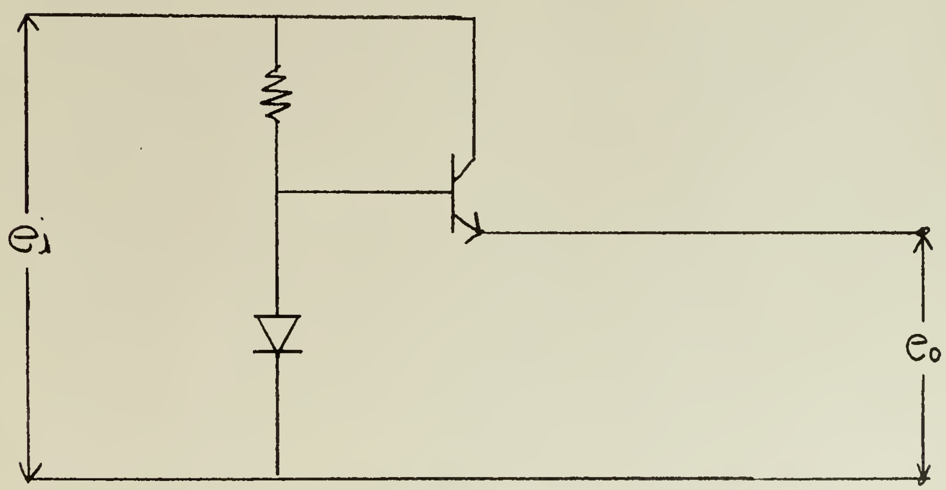

Figure 1.10 Simple Emitter Follower Voltage Regulator

closely than does the cathode follower makes it a more attractive regulator.

It has probably been noted that the analog to either the μ or g_m bridge type is absent. Transistor literature has been searched, to no avail, for a bridge circuit that would give similar operation. The author devoted a small amount of effort towards devising one. It is believed that if one is possible it would not be as useful as its vacuum tube counterpart because transistor small signal parameters display none of the constancy that μ and g_m do when the operating point is varied.

In the design of vacuum tube voltage regulators many practical difficulties combine to degrade their operation from that predicted by theory.(3)(7) Many of these are shared by transistor circuits. One exception is the introduction of ripple voltages by the filaments of the vacuum tubes in the regulator itself. The variation in performance due to temperature changes encountered in transistor circuits seems to offset this advantage. This problem, along with others, will be treated more fully in later sections.

It must be borne in mind that this paper is not advocating the complete replacement of vacuum tubes by transistors for voltage regulation. It is desired to point out, however, that for many applications the circuit designer should seriously consider the use of transistors. They often can give a most satisfactory performance with an accompanying reduction in size and complexity.

CHAPTER II

SERIES VOLTAGE REGULATORS

The basic concept of series voltage regulation is illustrated in Figure 2.1. The series element is a variable impedance whose function

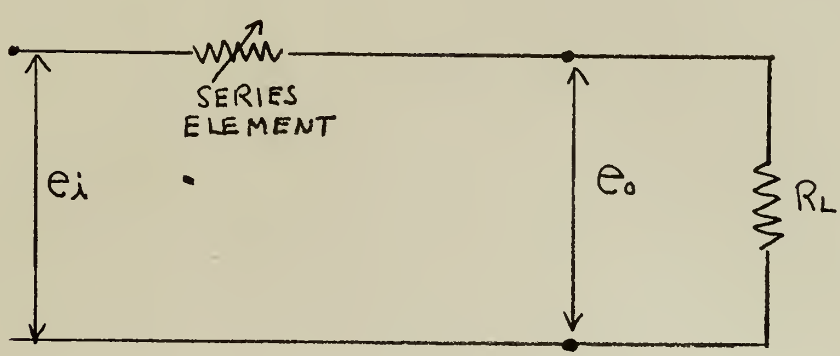

Figure 2.1 Concept of series regulation

is to change its value so that the output voltage is maintained constant despite input voltage and load current fluctuations. An active element, such as a vacuum tube or transistor, has the ability to act as such a variable impedance. To accomplish this the output voltage is sampled and compared with a reference element. The error that is developed is used to change the voltage between the terminals of the series element so as to reduce this error to zero.

Restricting ourselves to the use of a transistor as the series element there still remain numerous possible configurations for controlling its action. The two simple circuits illustrated in Figures 2.2 and 2.3 perform quite well as series voltage regulators. They take

advantage of the properties of transistors to perform their function and are not "transistorized copies" of tube circuits. These are considered basic because they employ the minimum number of both active and passive elements. Each will be examined in detail in this chapter. More complex circuits offering improved or modified performance will also be treated.

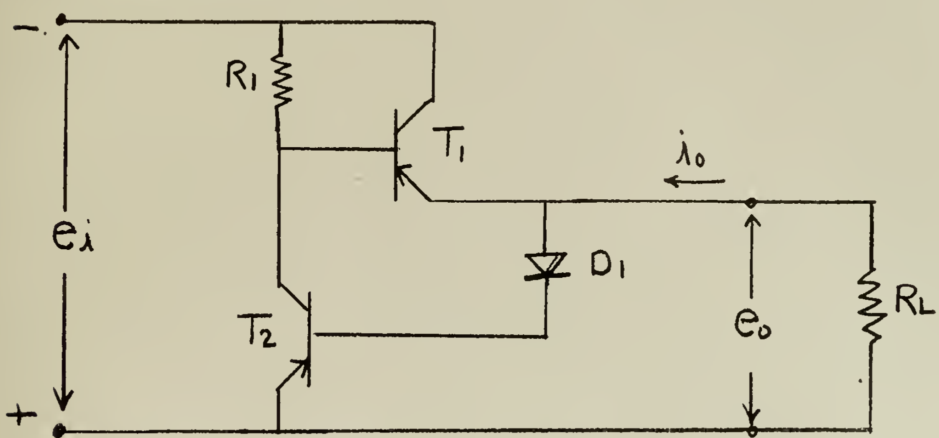

Figure 2.2 Symmetrical Transistor Series Voltage Regulator

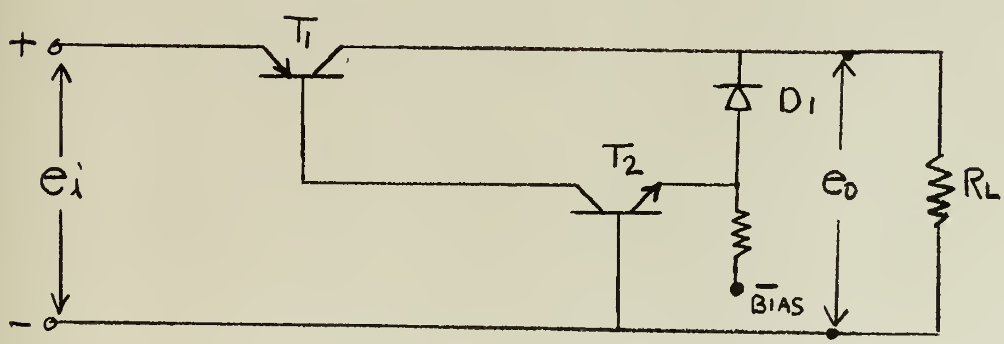

Figure 2.3 Complementary Transistor Series Voltage Regulator

Consider first the circuit using symmetrical transistors shown in Figure 2.2. Transistor T_1 is the series element while transistor T_2 and diode D_1 combine the functions of error detection and amplification.

D_1 is a silicon junction diode having a breakdown potential essentially the same as the desired output voltage. As the base to emitter potential of T_2 is in the order of 0.2 volts for germanium transistors and 0.5 to 1 volts for silicon units, it can be neglected for a first approximation. Qualitatively, the action is such that if the output voltage becomes more negative the base to emitter voltage of T_2 increases causing its collector current to increase. The resultant drop in the collector to emitter potential of T_2 makes the base of T_1 more positive. The result is an increase in the emitter potential of T_1 which will tend to restore the output voltage to its normal value.

In the complementary circuit of Figure 2.3, T_1 is the series element and D_1 has a breakdown potential nearly equal to the desired output. If the output voltage increases, the potential difference between the base and emitter of T_2 will decrease, thus decreasing emitter current. This in turn makes the collector of T_2 and the base of T_1 more positive therefore decreasing T_1's emitter current. The output voltage will tend to return to its original value.

The methods employed to analyze these circuits merit some discussion. As each embodies the principles of feedback, the works of Bode[8] and others were consulted in an attempt to simplify the analysis. The familiar feedback techniques find their greatest application where the active elements are in cascade and the feedback paths contain only passive networks. This is not the case in series regulators for the active elements are incorporated in the feedback path. Moreover, as Ghandi[9] has pointed out, the more elementary feedback theory that often can be applied to vacuum tube circuits is inapplicable to similar transistor

circuits. Briefly, the reasons are attributed to three points of difference between transistor and tube circuits: (1) Power is fed back rather than voltage; (2) The sampled power to be fed back appreciably loads the output; (3) The transistor must be considered bilateral over its entire frequency range. All these considerations deterred the employment of some modified feedback technique for analysis. The possibilities of approaching the problem by matrix techniques were investigated. This approach was ruled out because the manner in which the various four terminal networks are interconnected does not usually conform to the standard connections.[10][11][12] Straight mesh analysis was finally selected. However, unless circuits are kept fairly simple the resulting expressions become quite cumbersome and difficult to interpret.

The usual approach used to analyze vacuum tube regulators is illustrated by Hill.[2] It hinges upon the fact that in the frequency range of interest the vacuum tube is essentially unilateral. This is not appropriate for transistors. Nevertheless, Hill's definitions of the figures of merit for voltage regulators will be used. This approach results in simpler solutions and also divorces the regulator from both load and source.

The instantaneous output voltage of a power supply-regulator combination can be considered to be governed approximately by the linear relation

$$E_o + e_o = E_i + \mathcal{R} e_i - r i_o \, . \tag{2.1}$$

The voltage regulator regulation factor, $\mathcal{R}$, is defined by the

expression

$$\mathcal{R} = \frac{\partial e_o}{\partial e_i} \quad (2.2)$$

It is a measure of the regulator's effectiveness in minimizing input voltage variations. The voltage regulator output impedance factor, r , is defined as

$$r = -\frac{\partial e_o}{\partial i_o} \quad (2.3)$$

and expresses the worth of the regulator for combating varying load conditions. Ei and Eo are the nominal values of input and output voltage and need not be considered in regulator performance analysis. The entire system may then be represented in terms of variational components by Figure 2.4.

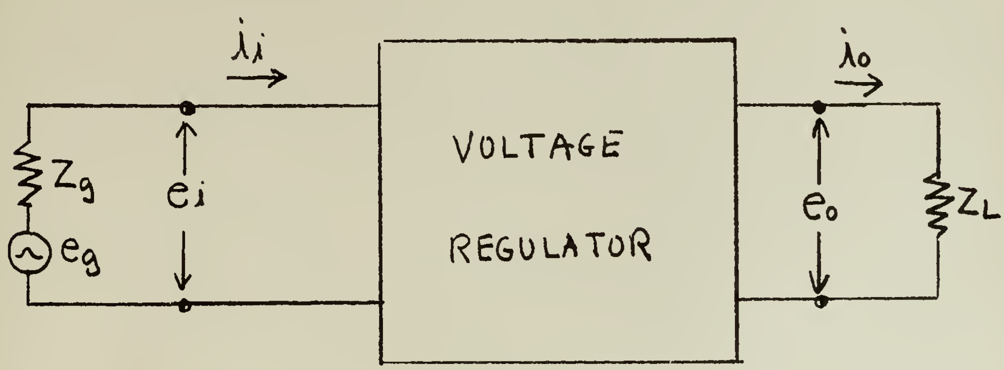

Figure 2.4 Equivalent Circuit for the Variational Components of a Regulator System

In order to derive expressions for the overall regulation factor and output impedance we can write from Figure 2.4

$$e_i = e_g - i_i Z_g . \quad (2.4)$$

Rewriting equation (2.1) in terms of variational components we have

$$e_o = \mathcal{R} e_i - r i_o . \tag{2.5}$$

Combining equation (2.4) and (2.5) results in

$$e_o = \mathcal{R} e_g + \mathcal{R} i_i Z_g - r i_o . \tag{2.6}$$

Hill now makes use of the fact that in most vacuum tube circuits i_o is nearly equal to i_i. It may be argued that this is not the case in the circuits we will be concerned with because transistors are by nature current operated devices. For purposes of discussion a current loss factor, k, will be defined by the relation

$$i_i = k i_o . \tag{2.7}$$

In some circuit configurations k is constant with a value nearly equal to the reciprocal of the grounded base current gain of the series transistor, $\frac{1}{\alpha}$. In other types of circuits k varies with load being a maximum under no load conditions. This is because certain bias currents are always flowing. At normal load k is in the order of $\frac{1}{\alpha}$ again. In either case the value of k can be obtained by inspection of the circuit. The derivation of the basic relations will be continued employing k to show its effect on performance. However for engineering purposes this current loss factor can usually be considered as unity.

Using equation (2.7) to modify (2.6) we can now write

$$e_o = \mathcal{R} e_g - i_o (\mathcal{R} k Z_g + r) . \tag{2.8}$$

This relation can be represented by the equivalent circuit of Figure 2.5

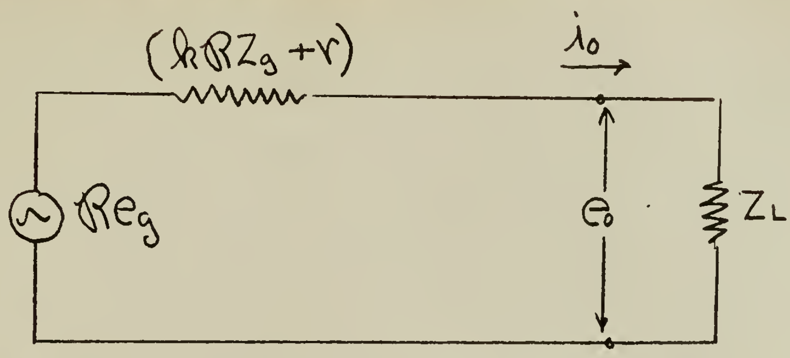

Figure 2.5 Simplified Equivalent Circuit for the Variational Components of a Regulator System

From the figure the expressions for the overall regulation factor and overall output impedance of the system may be written as

$$\text{overall regulation factor} = \frac{\mathcal{R}Z_L}{k\mathcal{R}Z_g + r + Z_L} \qquad (2.9)$$

and

$$\text{overall output impedance} = k\mathcal{R}Z_g + r \qquad (2.10)$$

For purposes of analysis $\mathcal{R}$ and r are solved for by circuit theory. In the frequency ranges of normal interest $\mathcal{R}$, r and k are pure numbers. As the frequency is increased various capacities and inductances inherent to the transistors and other components make these three complex. This paper will only consider the low frequency case.

By considering the relative magnitude of the terms of equations (2.9) and (2.10) these expressions may be simplified. In most practical regulators $\mathcal{R}$ is in the range from .01 to .001. The low frequency output impedance for practical series regulators is usually less than

ten ohms. For extremely small load currents the maximum conceivable value of k would be equal to the grounded emitter current gain of a transistor, or about 20. In this case Z_L would be very large anyway and the denominator of equation (2.9) would be nearly equal to Z_L itself. On the other hand at normal load where Z_L is small k is also quite small, being about $1/\alpha$, or 1.05, and again the denominator of equation (2.9) would be essentially Z_L. Therefore, unless the internal impedance of the unregulated source, Z_g, is quite large the overall regulation factor can be considered the same as $\mathcal{R}$.

For practical purposes we can often consider the overall output impedance to be identical to r. The validity of the approximation rests on the value of term $k\mathcal{R}Z_g$. The possible variations in k from essentially unity to about 20 have been pointed out. When a regulator is operating in the designed range though, the value usually is less than 1.5. If Z_g is of reasonable magnitude then the overall output impedance is very nearly r at normal loads. On the other hand if it is desired to predict performance at very low currents k must be considered in some classes of circuits. The author has neglected k for the analysis of all circuits without any loss in accuracy.

In work to follow, unless expressly stated to the contrary, the author will endeavor to use the same symbols as Shea[11]. The appropriate ones are redefined in the list of symbols. Two deviations that may produce confusion are the symbols α and β . The symbol α is usually defined as the grounded base short circuit amplification factor while "a" is defined as $\frac{rm}{rc}$. As "a" is very nearly equal to α , α will be used for both terms. This is quite common in the literature.

The term β is defined as the grounded emitter short circuit amplification. It is related to α by equation (2.11).

$$\beta = \frac{\alpha}{1-\alpha} \tag{2.11}$$

An additional term, R_D, is defined as the dynamic resistance of the reference diode.

Throughout the paper the following typical values for small signal parameters for low power transistors will be employed unless otherwise specified:

$$\begin{aligned} r_e &= 25 \text{ ohms} \\ r_b &= 500 \text{ ohms} \\ r_m &= 950 \text{ K ohms} \\ r_c &= 1 \text{ meg} \\ \alpha &= 0.95 \\ \beta &= 20 \\ R_D &= 100 \text{ ohms} \end{aligned} \tag{2.12}$$

These figures are quite conservative, but they will allow us to compare various circuits numerically. The variations of parameters between different transistors of the same type and their changes with operating point and temperature preclude the accurate prediction of results unless each unit is tested.

Let us now consider the symmetrical transistor regulator of Figure 2.2 in some detail. It is obvious that the best regulation may be obtained if R_1 approaches infinity. This is because input voltage variations will not be impressed upon the base of T_1. There-

fore the circuit of Figure 2.6 will be analyzed to determine the ultimate in performance.

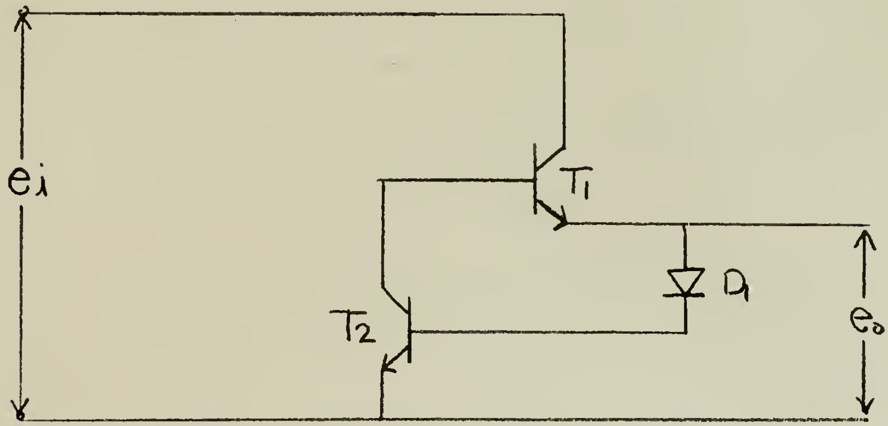

Figure 2.6 Ultimate Symmetrical Regulator

The expressions for r and $\mathcal{R}$ are derived in Appendix I. The simplified results are shown below:

$$r \approx \frac{(1-\alpha_1)\left[r_{e2} + (1-\alpha_2)(R_D + r_{b2})\right]}{1-\alpha_1(1-\alpha_2)} \qquad (2.13)$$

$$\mathcal{R} \approx \frac{r_{e2} + (1-\alpha_2)(R_D + r_{b2})}{r_{c1}\left[1-\alpha_1(1-\alpha_2)\right]} \qquad (2.14)$$

The factor k is not applicable here for there is no current path for the collector current of T_2. Substituting the values of equation (2.13) into the above expressions we obtain:

$$r \approx 2.9 \text{ ohms}$$

$$\mathcal{R} \approx .0000584.$$

A practical version of the symmetrical regulator must provide a path for the collector current of T_2 and the base current of T_1. The easiest solution is to insert the resistor R_1 as shown in Figure 2.2. Physical reasoning indicates that this will mainly affect $\mathcal{R}$. The alteration of the ultimate value of r will be a second order effect. An expression for the modification of $\mathcal{R}$ by R_1 has been derived in Appendix II. The simplified result is:

$$\mathcal{R} \approx \frac{(R_1 + r_{c1})\left[r_{e2} + (1-\alpha_2)(R_D + r_{b2})\right]}{r_{c1}\left\{R_1\left[1-\alpha_1(1-\alpha_2)\right] + r_{e2}\right\}} \tag{2.15}$$

The variation of $\mathcal{R}$ with R_1 is shown in Table 2.1

R_1	$\mathcal{R}$
1 K	.055
10 k	.0055
100 K	.00055
∞	.000058

TABLE 2.1

In order to verify the theoretical analysis, the circuit shown in Figure 2.7 was constructed. The magnitude of R_1 was limited by several practical considerations. It is approximately equal to $\frac{e_i - e_o}{i_1}$.

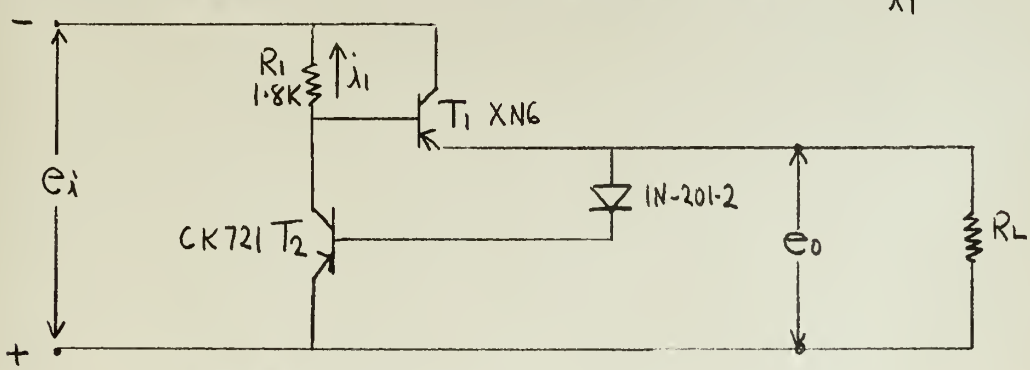

Figure 2.7 Practical Symmetrical Regulator

The current i_1 is constant, but its components i_{b1} and i_{c2} vary with load. The value of i_1 is chosen so that at full load i_{c2} is still large enough to permit T_2 and the diode to operate satisfactorily. In other words, the minimum i_{c2} should be of such a value as to keep T_2 out of the region where the term

$$r_{e2} + (1-\alpha_2)r_{b2}$$

is large. This is essentially the grounded base input resistance of T_2 and it appears in the numerator of the expressions for both r and $\mathcal{R}$. For a typical transistor the value of this term at one milliamp of collector current may be five times that at 5 ma. The minimum value of i_{b2} should be chosen so that the diode is operated above its noisy region. The allowable collector dissipation of T_1 usually limits the value of e_1.

This circuit belongs to the class that have a k dependent on load, The current i_{c2} does not reach the load. It has been pointed out above that i_{c2} is a maximum at no load and decreases as i_o increases. The circuit was tested and the results are shown in Figure 2.8. The fact that the curves are linear at low output current substantiates the validity of our decision to ignore k in predicting results. As a further test the output impedance was checked with frequency at several current levels. This approach was used in an attempt to unmask any averaging effect caused by DC measurements. The results are shown in Table 2.2. It can be seen that at extremely low currents k does degrade the output impedance somewhat. This effect disappears quite rapidly though. The output impedance of the unregulated source for this test

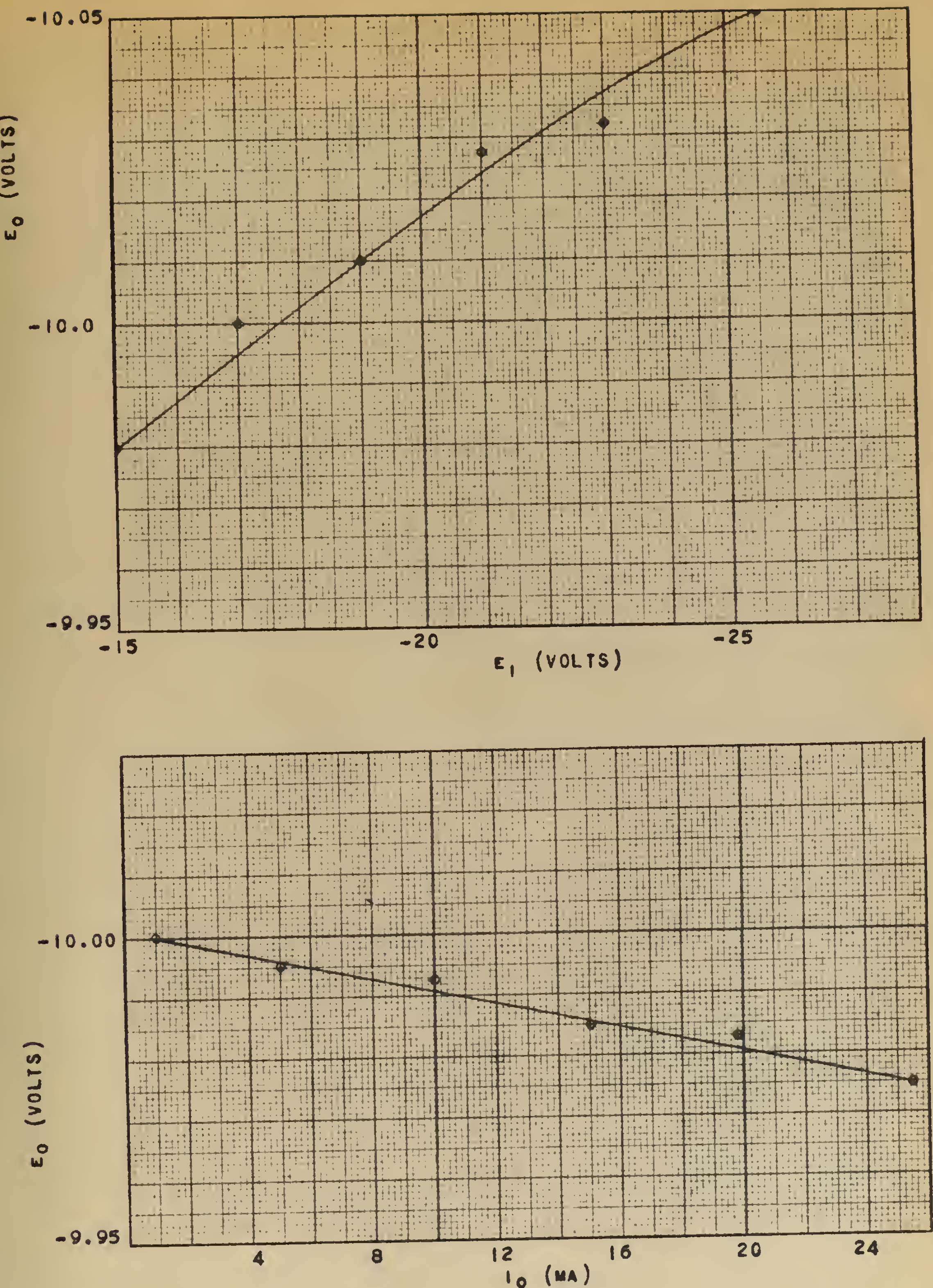

FIGURE 2.8 DC PERFORMANCE OF THE CIRCUIT OF FIGURE 2.7

was measured to be 92.5 ohms at 10 cycles.

FREQUENCY	LOAD CURRENT	OUTPUT IMPEDANCE
10 cycles	1 ma	2.22 ohms
10 cycles	3 ma	1.98 ohms
10 cycles	15 ma	1.98 ohms
1 KC	1 ma	2.22 ohms
1 KC	15 ma	1.8 ohms
10 KC	1 ma	2.22 ohms
10 KC	15 ma	1.8 ohms

TABLE 2.2

The overall regulation is .0076 and the DC output impedance is one ohm. This is somewhat better than predicted for both the ultimate r and the $\mathcal{R}$ of equation (2.16). This is because both the XN6 and the CK721 are better transistors than the typical one described in equation (2.12). The XN6 is an experimental germanium medium powered transistor manufactured by Motorola. The CK 721 is a commercially available audio transistor made by Raytheon. The IN-201-2 is a National Semiconductor silicon junction diode having a Zener breakdown potential of about ten volts. As stated before, the author believes that the worth of transistor small signal analysis lies mainly in predicting order of magnitude results. The temperature characteristics of this circuit are discussed in Chapter V. See Figure 5.7.

It was mentioned in the previous paragraph the i_{b1} must be sufficient to keep the diode in its most satisfactory operating region. For typical silicon junction diodes this means that the current should be greater than 200 μ amps. Figure 2.9 indicates a method of providing a bias current for this purpose rather than relying upon the base current of T_2. An analysis of R_2's effect upon the ultimate r appears in

Appendix III. Calculated results are shown in Table 2.3. Experimental results for several combinations of R_2 and E_2 appear in Table 2.4

R_2	r
0	124
10	27.9
100	.0
1000	3.32
10 K	2.92
∞	2.88

TABLE 2.3

R_2	E_2	DC Output Impedance
1.7 K	0	1.7
47 K	45 V	1.6
100 ohms	0	1.8

TABLE 2.4

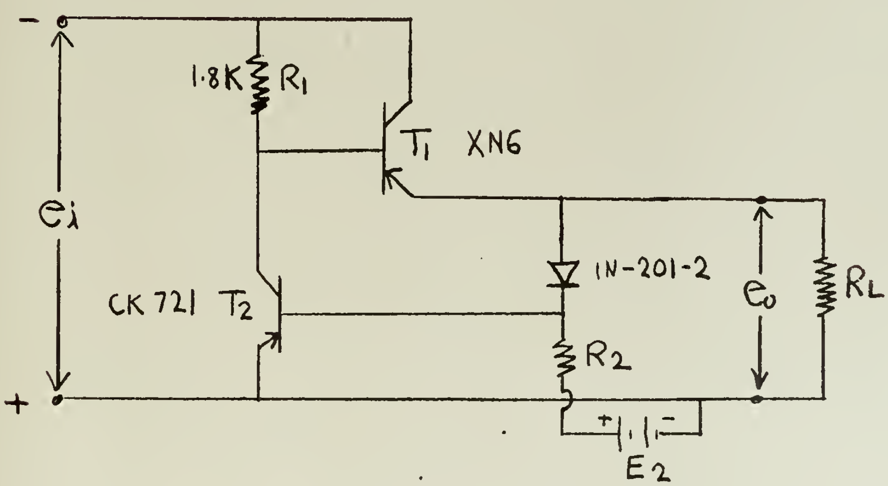

Figure 2.9 Method of Providing Diode Bias Current

In most practical applications it would probably be most convenient to omit E_2. Assuming 0.2 volts as a typical base to emitter drop for T_2, then R_2 could be made around two kilohms. This provides adequate diode current in itself so i_1 could be chosen upon T_2 considerations only. This will permit a slight increase in R_1 and consequently an improvement in regulation. This method should not be used for germanium transistors where large temperature variations are to be expected. The reason is that their base to emitter voltage decreases rapidly with temperature and may even reverse its sign. This effect is not as pronounced in silicon units until temperatures greater than 150°C are attained.

To effect a substantial improvement in regulation it will be necessary to add another active element. Examining the problem qualitatively, the reason that a small R_1 gives poor regulation can be seen by considering R_1 and T_2 as a voltage divider. The fraction of the input voltage variations that appear upon the base of T_1 is substantially inversely proportional to R_1. A method to keep the percentage of input variations at the base of T_1 small and still permit adequate current flow through T_2 is shown in Figure 2.10. It is reasoned that

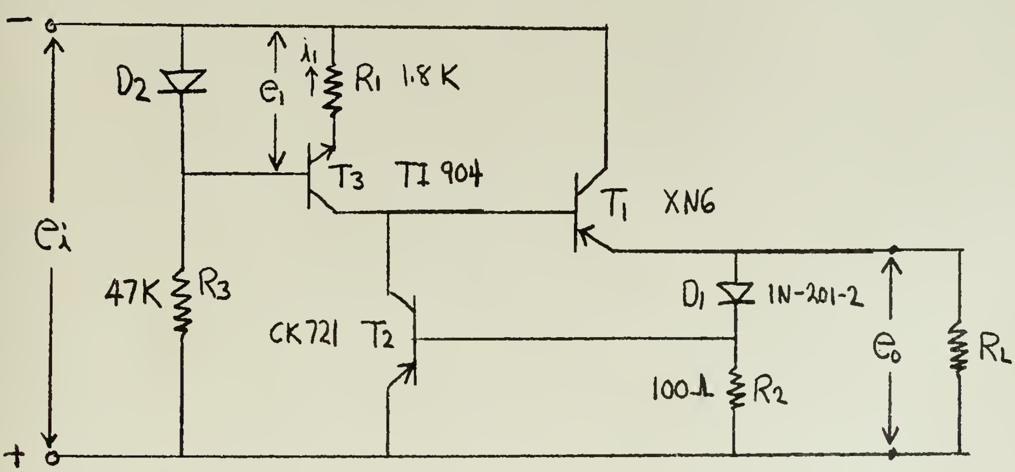

Figure 2.10 Improvement Upon Symmetrical Series Regulator

if i_1 can be maintained constant with input voltage changes then i_{e2}, and consequently the voltage at the base of T_1, will remain constant. The junction diode maintains e_1 constant and therefore input variations appear as changes in V_{cb} of T_3. By choosing R_1 so that T_3 operates on that portion of its characteristic curve where its collector current is independent of collector voltage we can achieve the desired constant i_1. The circuit was constructed using the values indicated. However, a 4.5 volt battery was used in place of D_2 because a suitable diode was not available. Using the rule of thumb that the dynamic output impedance of a battery is about five ohms per cell the battery has an R_D of about 15 ohms. This is probably greater than the R_D of a 4.5 volt reference diode. The TI 904 is a low power silicon NPN transistor made by Texas Instrument, Inc. Its β is about 20. When tested the output voltage changed .005 volts for an input variation of 12 volts. The regulation then is .00042. These measurements were taken using a differential technique with a Millivac Type MV-17C DC Voltmeter. The DC output impedance of 2.3 ohms is materially the same as in previous circuits. The variations of output impedance with frequency are shown in Figure 2.11. This circuit was chosen for this test because it is the final modification to the basic symmetrical regulator.

The output impedance of the circuit of Figure 2.10 is usually more than adequate to meet even laboratory requirements. However, if it is desired to reduce its value further, compensation methods, as indicated in Figure 2.12, are available. This particular circuit is identical with that of Figure 2.9 except for the addition of the small

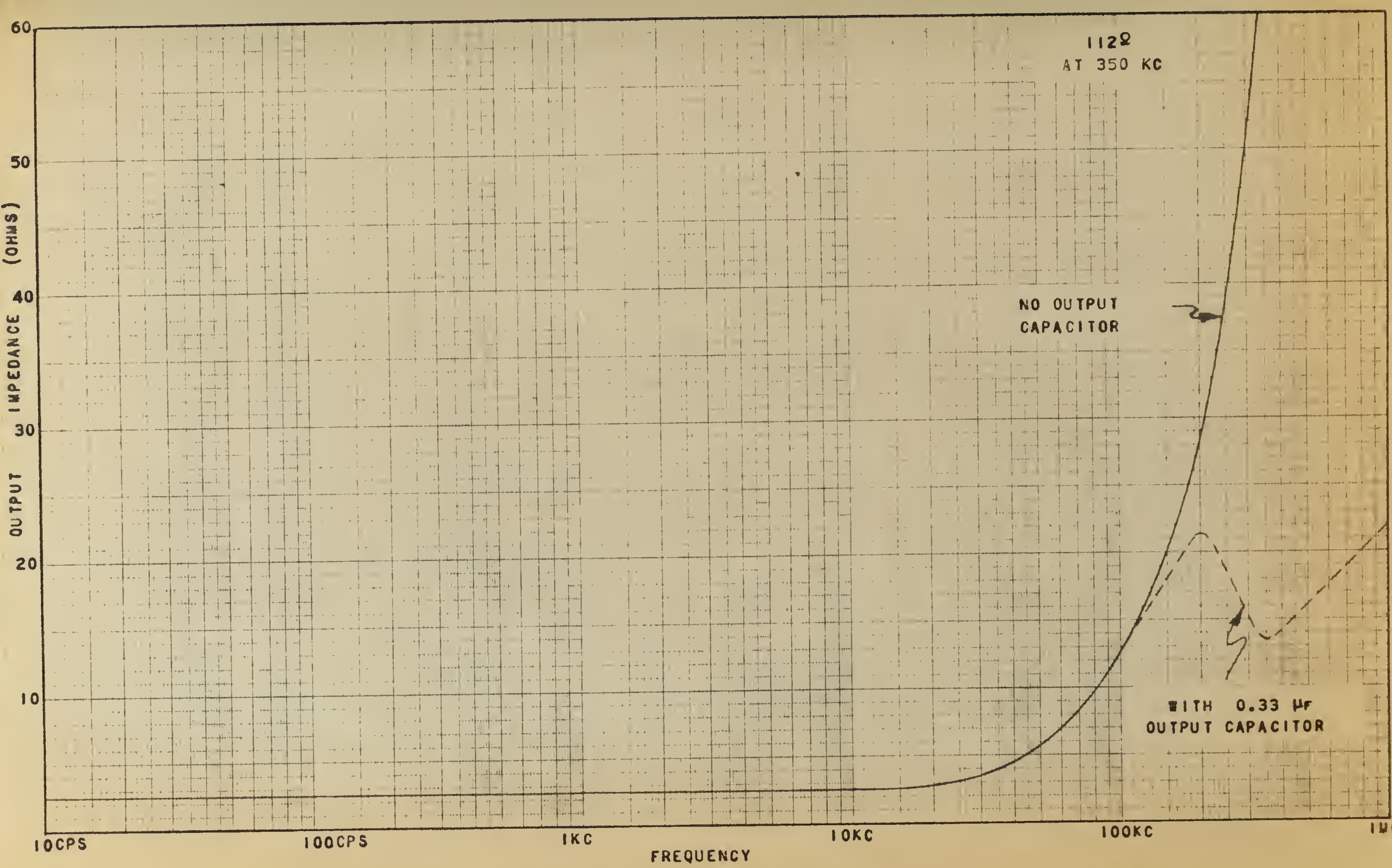

FIGURE 2.11 OUTPUT IMPEDANCE VARIATIONS WITH FREQUENCY FOR CIRCUIT OF FIGURE 2.10

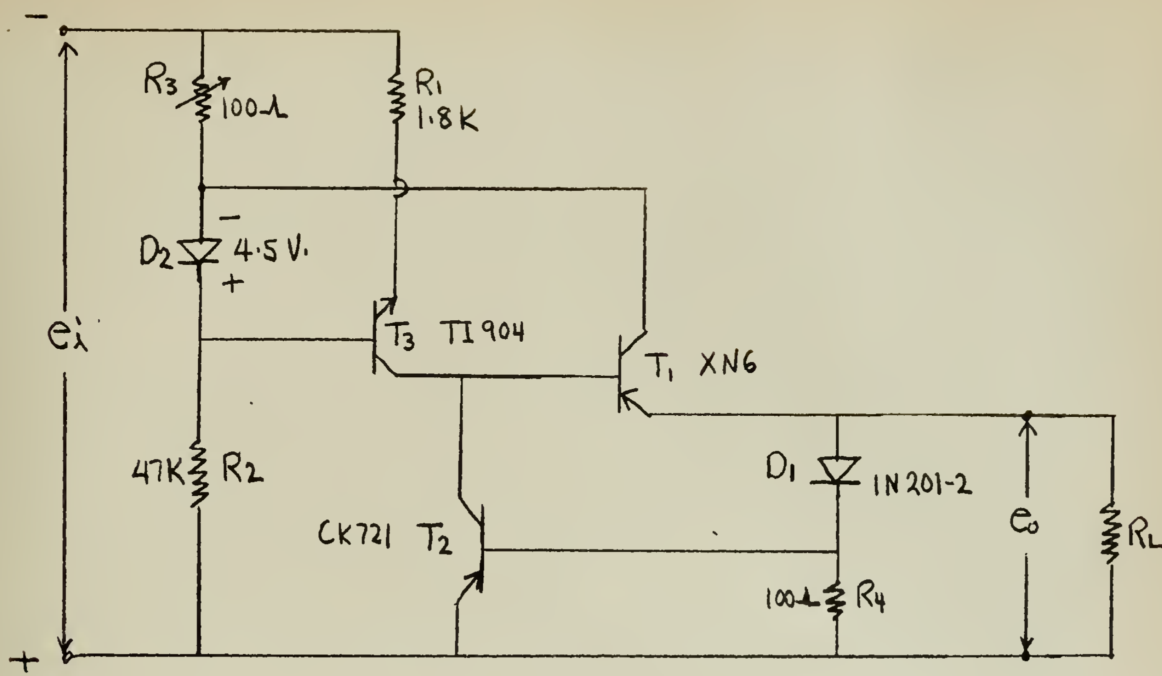

Figure 2.12 Regenerative Compensation to Reduce Output Impedance

resistor R_3. The major factor contributing to the output impedance of Figure 2.10 is the decrease in current through D_1 as the load current is increased. The resistor R_3 uses regeneration to compensate for this. Now, if i_o increases so does the voltage drop e_1 across R_3. This will result in an increase in i_{e3} and consequently in i_{c2} and i_{b2}. If the value of R_3 is chosen properly, the current through D_1 will remain essentially constant despite variations in load. A quick method for determining the approximate magnitude of R_3 for perfect compensation can be obtained by equating expressions for the two voltage changes due to load variations. Now

$$i_o r \approx i_o \frac{R_3}{R_1}\left[r_{e2} + (1-\alpha_2)(r_{b2} + R_D)\right],$$

but

$$r \approx (1-\alpha_1)\left[r_{e2} + (1-\alpha_2)(r_{b2}+R_D)\right]$$

Combining these two relations we have

$$R_3 \approx (1-\alpha_1)\, R_1 \qquad (2.16)$$

A value for R_3 of 85 ohms was calculated using equation (2.16). The circuit, with component values as shown, was tested. The measured value of R_3 for zero output impedance was 94.6 ohms. The regulation was substantially the same as before. Increasing R_3 above this figure results in a negative output impedance which may be desirable under some circumstances. Unless it is absolutely mandatory to have such a low output impedance, compensation is not recommended because of the danger of oscillations. Nevertheless, similar compensation techniques are applicable to most series and emitter follower type regulators.

Let us now consider the other basic form for a series regulator introduced in Figure 2.3. A practical circuit is shown in Figure 2.13 for convenience. The theoretical analysis of this circuit is contained in Appendix IV. The resulting expressions for $\mathcal{R}$ and r are repeated below.

$$r \approx \frac{(1-\alpha_1)\left[r_{e2} + R_D + (1-\alpha_2)\, r_{b2}\right]}{1-\alpha_1(1-\alpha_2)} \qquad (2.17)$$

$$\mathcal{R} \approx \frac{R_D + r_{e2} + (1-\alpha_2)\, r_{b2}}{r_{c1}\left[1-\alpha_1(1-\alpha_2)\right]} + \frac{r_{b2} + \alpha_1(r_{e2}+R_D)}{r_{c2}\left[1-\alpha_1(1-\alpha_2)\right]} \qquad (2.18)$$

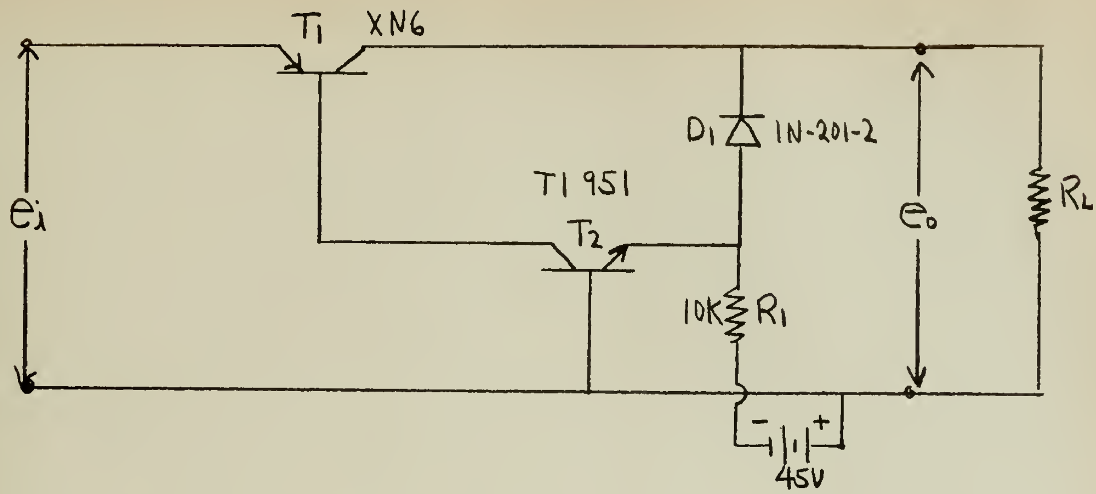

Figure 2.13 Practical Example of the Complementary Transistor Series Regulator

The current loss factor of this circuit is constant with load variations. It is essentially equal to the reciprocal of α_1. As k then is of the order of 1.05, it can be ignored completely. When the figures of equation (2.12) are substituted into these expressions we obtain values for $\mathcal{R}$ and r of .008 and 7.9 ohms respectively. It is noted from equation 2.13 that the output impedance is inherently inferior to that of the previous circuit. This is because of the difference in the connections for T_2 in the two circuits. On the other hand, the regulation is superior to that obtainable in the practical two transistor version of the symmetrical circuit.

The circuit of Figure 2.13 was constructed and tested. The TI 951 is a medium power silicon transistor manufactured by Texas Instruments. It was employed because it can withstand the large collector voltages anticipated. The entire input potential is impressed on the collector of T_2 in this configuration. The value of 10 K was chosen

for R_1 as a compromise between a reasonable load for T_2 and a satisfactory current range for the diode. The DC output impedance was measured to be 3.6 ohms and the overall regulation was 0.001. The discrepancy between this output impedance and the one calculated is attributed to lower values for r_b and R_D than those of equation (2.13). The poorer regulation is caused by the fact that the r_c of T_1 is small. This offsets any improvement resulting from reduced R_D, r_{b2} and r_{e2}. This particular XN6 was measured and found to have an r_c of only 130 K. The variation of output impedance with frequency are shown in Figure 2.14.

The complementary transistor series regulator has superior regulation but poorer output impedance than the two-transistor symmetrical series regulator. It should be noted, however, that a bias supply is mandatory for this circuit. This is because, under operating conditions, the emitter of T_2 is below ground potential by the amount of its base to emitter drop.

The reader has probably noted the absence of means for varying the output voltage in the circuits that have been presented so far. There are methods for including this feature, but each sacrifices circuit performance to do so. Vacuum tube regulators, on the other hand, can furnish a variable voltage output with ease and with very little degradation of performance. The reasons for this lie in the nature of the two active elements themselves. A vacuum tube under Class A conditions is a voltage operated device. The transistor, on the other hand, is both current and voltage operated. The voltage divider across the output that is the basis for most trimming schemes for vacuum tubes is still

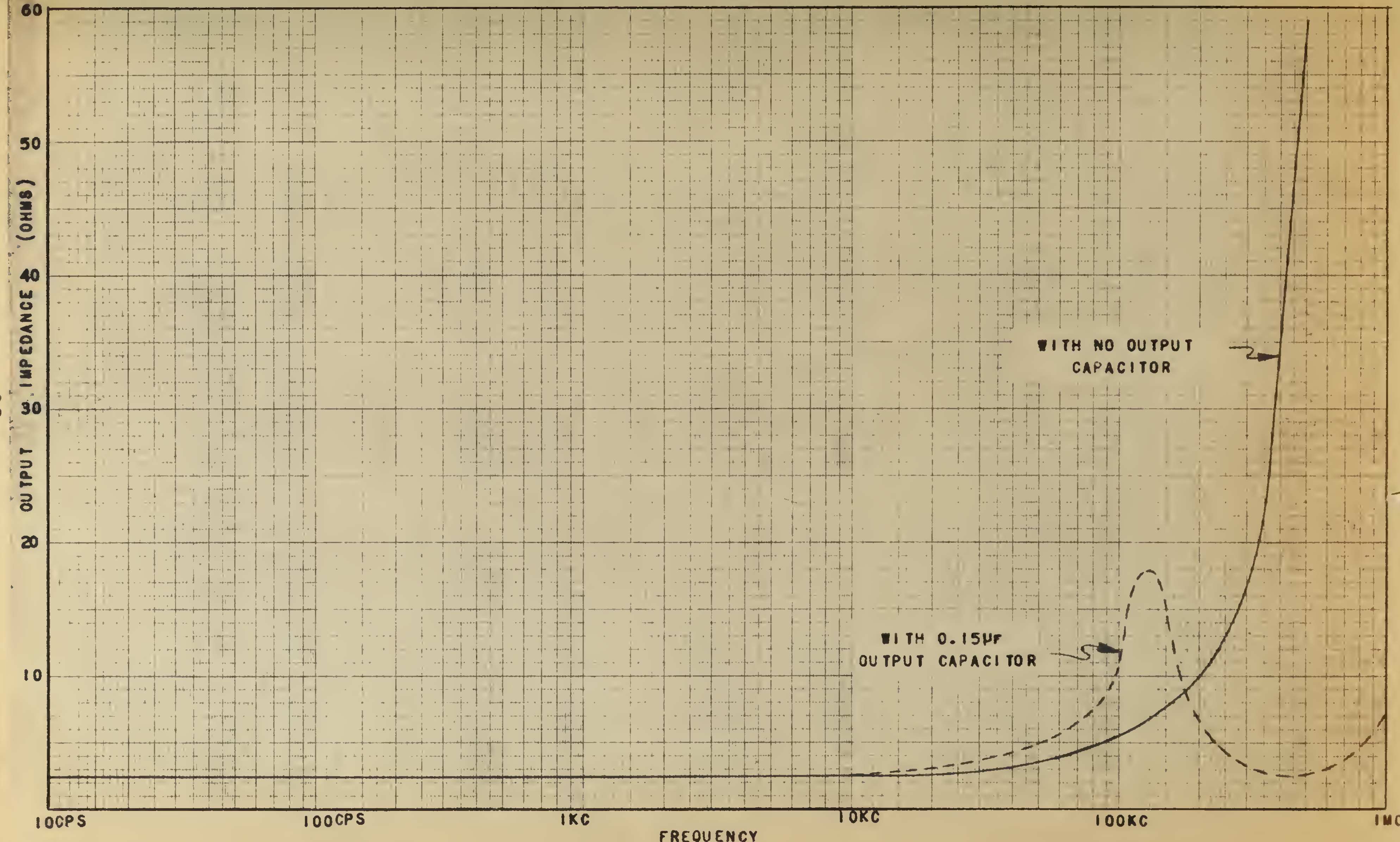

FIGURE 2.14 OUTPUT IMPEDANCE VARIATIONS WITH FREQUENCY FOR CIRCUIT OF FIGURE 2.13

used, but it now has an undesirable feature. It is impossible to keep load current variations from effecting its pick-off voltage. In Figure 2.5 (a) the voltage E' is a fixed fraction of E under Class A conditions. In Figure 2.15 (b) this is no longer true because i_2 is finite. This is

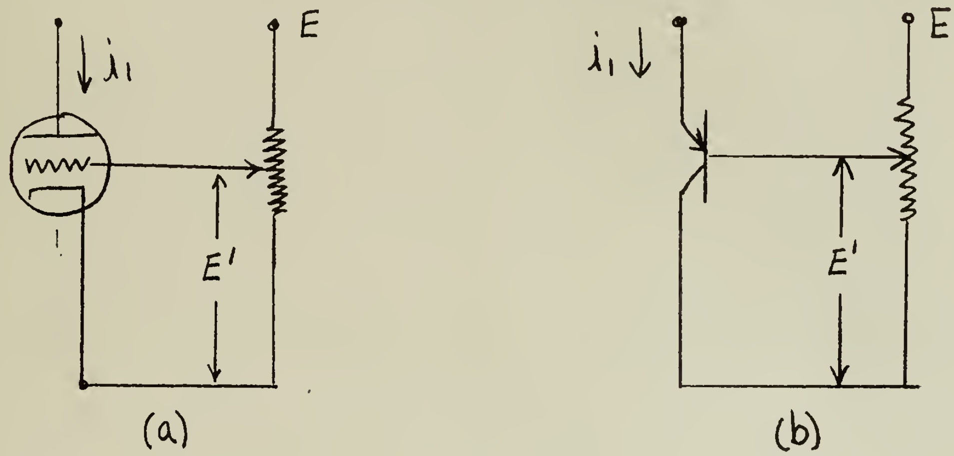

Figure 2.15 Comparison of Voltage Division

the heart of the trimming difficulties with transistors. There are several complicated schemes by which i_2 can be effectively reduced to zero. However, the author firmly believes that to take full advantage of the characteristics of the transistor, trimming should be omitted.

The circuit shown in Figure 2.16 was suggested by the authors of Reference (13). It is the exact duplicate of the most common form of vacuum tube series regulator except that signals are injected in the base of a transistor rather than in the grid of a tube. If the output voltage goes more negative, i_{e2} increases because of the increased emitter to base differential. This in turn will increase i_{e1} thus

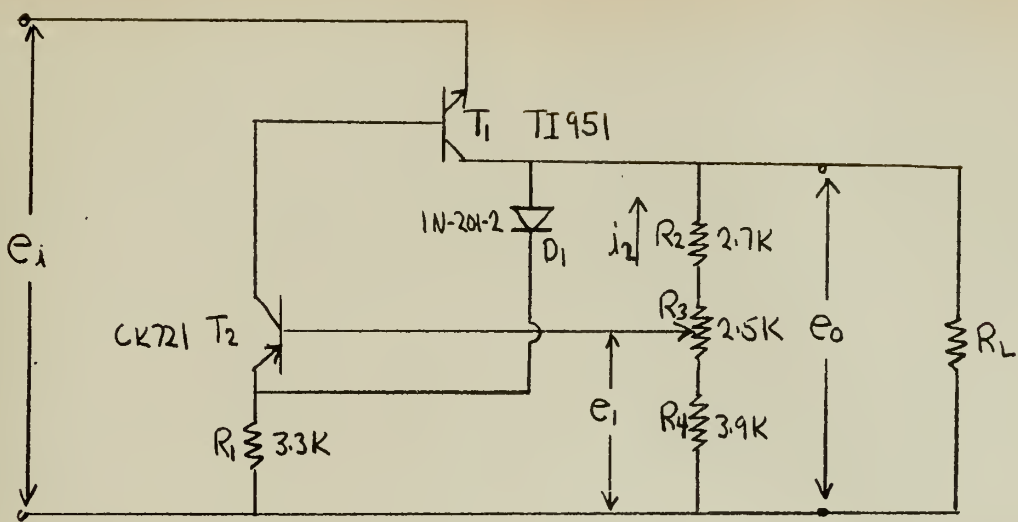

Figure 2.16 A Method of Trimming a Series Voltage Regulator

tending to return the output voltage to its normal value. The output voltage can be varied over an extent limited by the operating range of the transistors. In this circuit it was made variable from 17 to 23 volts. The value of R_1 is chosen so that there is still ample diode current at full load for the lowest output voltage setting. The resistance of the voltage divider branch should be kept quite low so that i_{b_2} is a small percentage of i_2. This will minimize the changes in e_1 due to load current variations. Table 2.5 indicates the regulation and output impedance at three voltage settings.

e_o	$\frac{\partial e_o}{\partial \lambda_o}$	$\frac{\partial e_o}{\partial e_i}$
-18	10.3	.009
-20	12.1	.0106
-22	14.4	.0112

Table 2.5

It is interesting to note that when R_2, R_3 and R_4 were increased by a factor of four that the output impedance was 25 ohms at 18 volts. This gives an indication of the effect i_{b_2} can have upon output voltage. The degradation of performance at higher output voltage also points this out. This is because that portion of R_3 associated with R_2 is increased at higher voltage settings.

The regulator of Figure 2.16 is an appropriate one with which to conclude this chapter for it points out that transistors are not just small triodes. In order to realize the worth of transistors, circuits should be designed to take advantage of their particular characteristics. The circuits previously discussed have indicated that nearly perfect performance can be obtained with relatively simple configurations. These regulators can satisfy the most exacting requirements within the operating range of the transistors and junction diodes.

CHAPTER III

THE SHUNT VOLTAGE REGULATOR

The most basic shunt voltage regulator is illustrated in Figure 3.1.

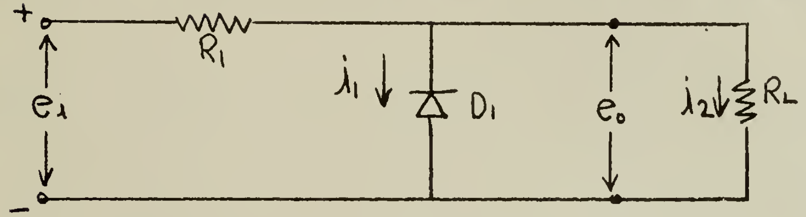

Figure 3.1 Simple Shunt Voltage Regulator

The action is quite simple. The constant voltage characteristics of the silicon junction diode fixes the output potential. If the output voltage rises, i_1 will increase and the resultant increase in the voltage drop across R_1 will return the output towards its original value. The regulation would be perfect if the diode were truly a constant voltage element.

The approach to be used for analyzing the effects of the finite resistance of the diode reference element on regulation was suggested by Berg(14). His derivation is repeated to that the validity of an extension will be appreciated.

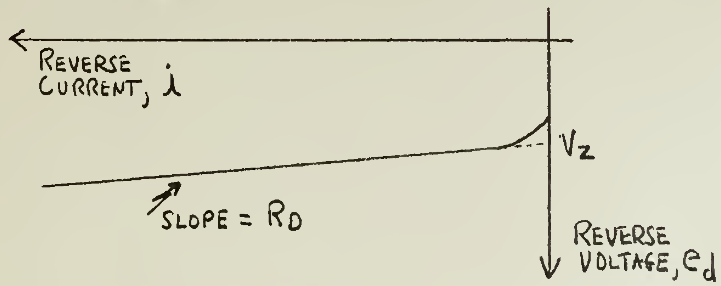

Figure 3.2 Reverse Characteristics of a Silicon Junction Diode

That portion of the silicon junction diode's characteristic which is used for a voltage reference is shown in Figure 3.2. It can be closely approximated by the equation

$$e_d = V_Z + i R_D \quad . \tag{3.1}$$

From Figure 3.1 we may write

$$\left.\begin{aligned} e_i &= (i_1 + i_2) R_1 + e_o \\ i_2 &= \frac{e_o}{R_L} \\ e_o &= e_d = V_Z + i_1 R_D \end{aligned}\right\} . \tag{3.2}$$

Combining the members of equation 3.2 we have

$$e_i = \left(\frac{e_o - V_Z}{R_D}\right) R_1 + \frac{e_o}{R_L} R_1 + e_o ,$$

which when solved for e_o results in

$$e_o = \frac{e_i R_D R_L + V_Z R_1 R_L}{R_1 R_L + R_1 R_D + R_D R_L} \tag{3.3}$$

Differentiating equation (3.3) with respect to e_i we obtain

$$\frac{de_o}{de_i} = \frac{R_D R_L}{R_1 R_L + R_1 R_D + R_D R_L} \equiv \mathcal{R}' \tag{3.4}$$

By combining equations (3.3) and (3.4) we have

$$\mathcal{R}' = \frac{e_o}{e_i + V_Z\left(\frac{R_L}{R_D}\right)} \tag{3.5}$$

This expression is quite useful in design work. This particular type of analysis was chosen, rather than the one used in Chapter II, because it led to the neat form of equation (3.5). $\mathcal{R}'$ is a measure of the worth of the system in stabilizing the output against variations in input voltage. It is closely allied to $\mathcal{R}$, as defined in equation (2.2), but not equal to it. The output impedance of this basic circuit is simply the parallel combination of R_D and R_1.

The circuit of Figure 2.1 is often a quite satisfactory answer to a regulation problem. It does possess several disadvantages. First of all it is quite wasteful of power. Equation (3.5) shows that for the circuit to be effective in minimizing input variations, $\bar{e}_o$ and R_1 must be large. In effect we sacrifice power for regulation. This fault is common to all shunt regulators. There are numerous instances where this is acceptable, and the shunt type circuit is widely used. One particular situation where the shunt regulator is usually the most advantageous type is where a low voltage bias supply is needed and a higher voltage source is already available in the equipment. The savings in space, weight and complexity gained by the shunt regulator over a rectifier, plus a more efficient regulator, will usually more than compensate for the power loss.

Two other shortcomings of the basic shunt circuit are the finite value of R_D and the fact that all load current decreases result in an equal increase in diode current. Unlike the inherent power loss, these two disadvantages may be minimized by the addition of one or more transistors. Figure 3.3 indicates the basic method for doing this. Neglect R_2 for the moment. The transistor now absorbs most of the load

current variations and the current flow through the diode reference is reduced by a factor of β . In addition this current gain reduces the effective value of R_D.

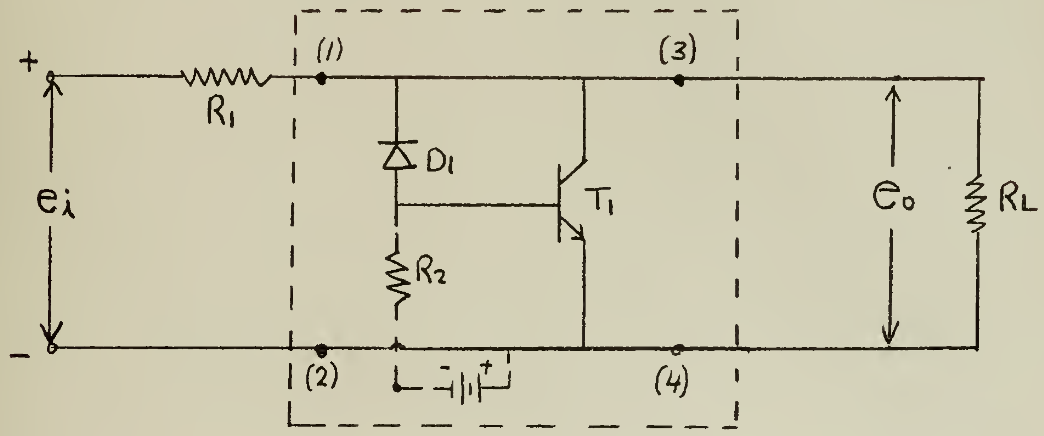

Figure 3.3 Basic Transistor Shunt Regulator

The performance of the circuit of Figure 3.3 can be determined by considering that portion enclosed within the dashed rectangle as an improved diode having an effective dynamic resistance of R_D'. To find R_D' we solve for the input impedance at terminals (1) and (2) with (3) and (4) open-circuited. The derivation appears in Appendix V. The resulting simplified expression is

$$R_D' = r_e + (1-\alpha)(R_D + r_b) . \tag{3.6}$$

As could be expected, this is essentially the input resistance to a grounded emitter stage plus $1/\beta$ times R_D'. Using equation (3.6) we can write the expression for the voltage regulation as

$$\mathcal{R}' = \frac{e_o}{e_i + V_z\left[\dfrac{R_1}{(1-\alpha)(R_D + r_b) + r_e}\right]} . \tag{3.7}$$

The derivation for r, the voltage regulator output impedance factor, also appears in Appendix V. The value

$$r = \frac{R_1 \left[r_e + (1-\alpha)(R_D + r_b) \right]}{R_1 + r_e + (1-\alpha)(R_D + r_b)} \qquad (3.8)$$

is the expected parallel combination of R_1 and R_D' . In practical circuits r can usually be considered equal to R_D'. The power transistors usually employed for these circuits have small signal parameters differing from those of equation (2.12). Typical values for germanium units are

$$r_b = 30 \text{ ohms}$$

$$r_c = 2 \text{ ohms}$$

$$\beta = 20.$$

Using these, and an R_D of 100 ohms, R_D' would be under ten ohms. This is more than adequate for most applications.

The shunt regulator presently possesses another practical advantage. The normal method to convert a positive supply to a negative one is to substitute transistors of the opposite type, NPN for PNP, or vice versa. At the time of writing only one manufacturer has been able to put PNP silicon transistors on the market, and these are low power units. For work at elevated temperatures silicon transistors are mandatory. The shunt type circuit is unique, to the writer's knowledge, in that it can be simply converted to give both a negative and positive supply using the same transistor type. The worth of this feature will undoubtedly diminish in the future as silicon transistor manufacturing techniques are improved. Nevertheless, the author has found that this advantage

has dictated the use of shunt regulators in several circuits developed by him.

The modified shunt regulator is illustrated in Figure 3.4. The

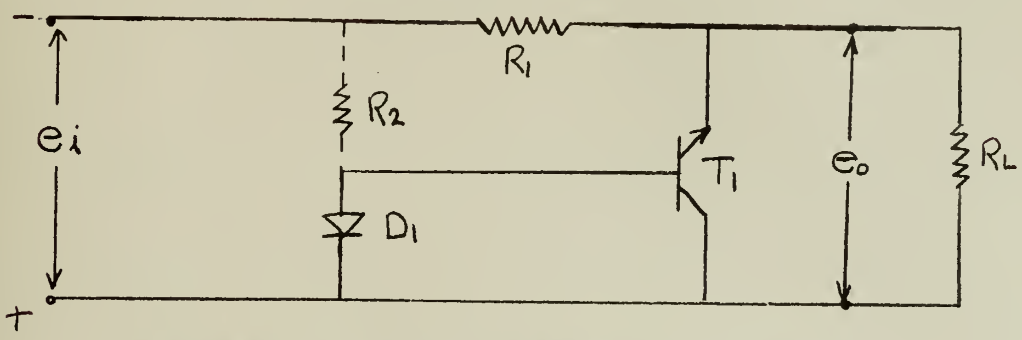

Figure 3.4 Modified Transistor Shunt Regulator

operation is identical to that of the original circuit in Figure 3.3. The only differences occur when it is necessary to provide a bias current to the diode in order to maintain it in the optimum current range. The resistors, R_2, shown dashed in both Figure 3.3 and Figure 3.4, indicate the two methods employed.

A number of both types of circuits were tested using various transistors and voltage levels. The circuit of Figure 3.5 is an example of the type introduced in Figure 3.4. The XN6 has a fairly low grounded base input impedance which makes it attractive for this circuit. This particular IN-201-2 was tested and found to have an R_o of 56 ohms and a V_z of about 9.8 volts. The value of R_1 was chosen so that at full load current there would still be approximately two milliamps of current through the transistor thus maintaining it in a satisfactory region. This is sufficient to keep the grounded base input impedance reasonably small at full load. The desire to have at least 0.2 ma of current

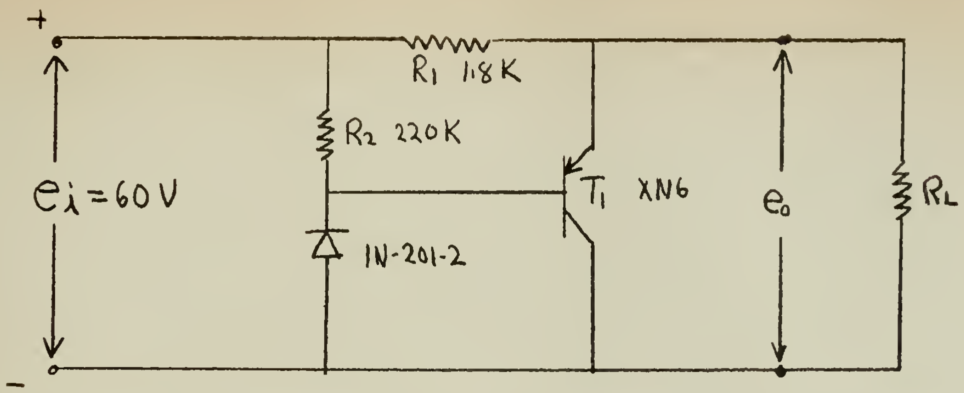

Figure 3.5 Example of Modified Transistor Shunt Regulator

through the diode at full load dictated the value of R_2. It can be seen that actual circuit design is quite simple.

A test of this circuit showed that the output voltage changed 0.6 volts as the load current was varied from 5 to 22 ma. This indicates an output resistance of about 3.5 ohms for DC variations. The regulation was measured to be .0029 for voltage changes ten volts either side of the nominal 60 volt input. This particular transistor was damaged before its parameters were measured, but the validity of the design equations can be checked by using the measured value of output impedance as R_D in equation (3.5). The calculated $\mathcal{R}'$ of .002 is in agreement with the measured value of .0029. These results are considered typical for germanium medium power transistors.

The other configuration is illustrated in Figure 3.6 by a circuit employing a Texas Instrument type 970 silicon power transistor. No biasing

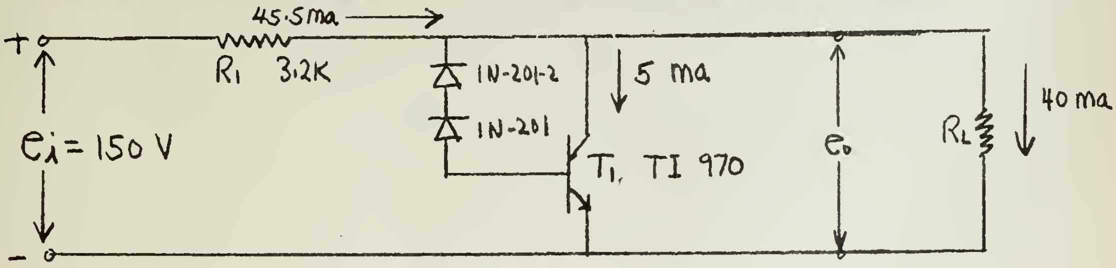

Figure 3.6 Example of Transistor Shunt Regulator

resistor was used because at full load a diode current of 0.5 ma was expected. This is because the TI 970 has a β of about ten and requires several milliamperes of current for good operation. The currents indicated in each branch are for full load conditions. This circuit was tested quite thoroughly as a similar one was to be included in a piece of equipment designed by Motorola. DC output impedance and regulation were measured at room temperature and at 105° C. The values were nearly the same, $\mathcal{R}'$ being about .0048 and output impedance 16.7 ohms. The constancy with temperature is expected because both β and input impedance increase with temperature. The fact that the parameters of the TI 970 are not as good as those of the XN6 account for the poorer performance. The reader must realize though that the TI 970 is the first commercially available silicon power transistor. The output voltage increased 0.78 volts as the temperature was raised from 37° C to 108° C, other conditions remaining constant. It is interesting to note that the change in reference voltage of the two diodes was previously measured to be 1.05 volts with the same temperature variation. The improvement can be attributed to the temperature coefficient of the emitter-to-base voltage being of opposite sign to that of the diodes. This phenomena is discussed in Chapter V. The results of a test of output impedance at various frequencies are shown in Figure 3.7. It can be seen that the addition of a 0.15 μf capacitor across the output will keep the output impedance reasonable throughout the frequency range of normal interest.

For applications requiring large load current variations the base current of the shunt transistor may reach values that will exceed the rating of the diode. This is especially true when using the TI 970 as

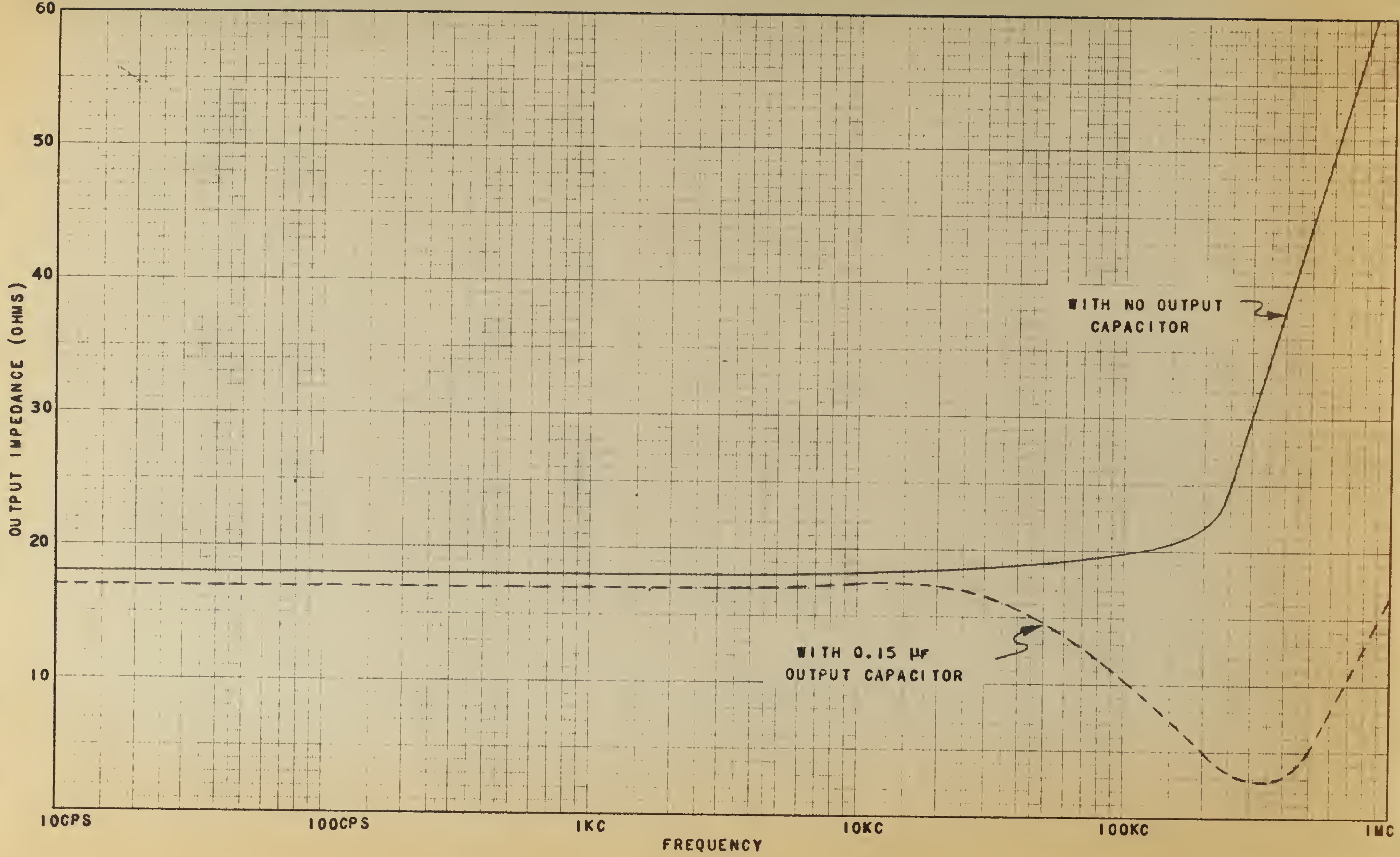

FIGURE 3.7 OUTPUT IMPEDANCE VARIATIONS WITH FREQUENCY FOR THE CIRCUIT OF FIGURE 3.6

values of β as low as seven have been noted. A circuit to overcome this difficulty is shown in Figure 3.8. Resistors R_2 and R_3 may be used

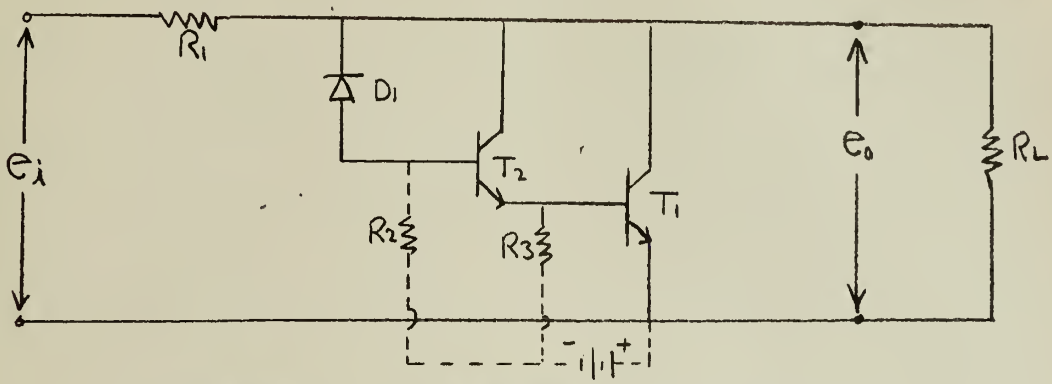

Figure 3.8 Two Transistor Shunt Regulator

when it is necessary to provide a bias current to the diode or to raise I_{c2} above I_{c_0}. The main purpose of T_2 is to reduce the current flow through the diode by a factor of β_2. It does result in performance slightly better than the one transistor circuit, but usually not enough to warrant the extra unit. By reasoning similar to that employed for the single transistor circuit it can be seen that

$$R_D' \simeq \frac{1}{\beta_1 \beta_2}\left(R_D + r_{b2}\right) + \frac{1}{\beta_1}\left(r_{e2} + r_{b1}\right) + r_{e1} \cdot \tag{3.9}$$

Output impedance and regulation may be calculated by substituting equation (3.9) into the previous formulas.

A practical two transistor circuit of the modified type appears in Figure 3.9. Resistor R_2 provides a minimum of 0.2 ma for the diode while R_3 insures that i_{c_2} is always greater than one milliamp. The output impedance was measured to be about three ohms and the regulation .0023. The reason for the rather small improvement over the circuit

of Figure 3.5 is that we must still live with r_{e1}. It is obviously the largest contributor to R_D' .

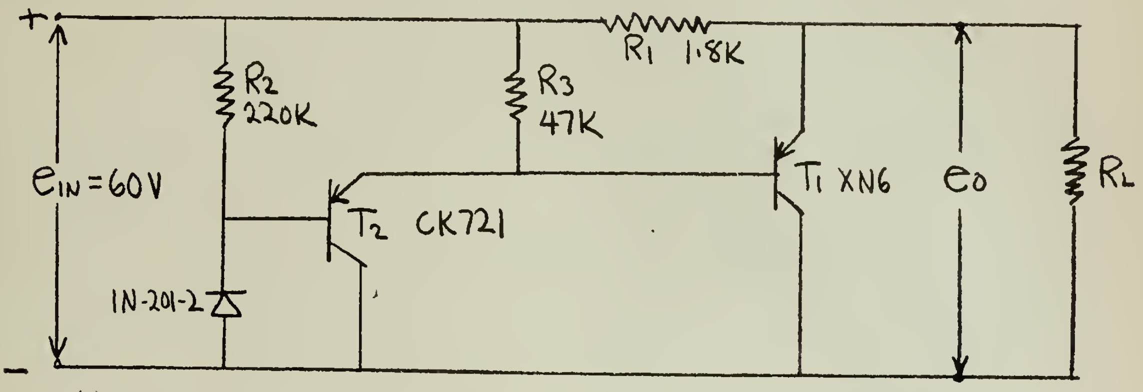

Figure 3.9 Modified Two Transistor Shunt Regulator

The other form of the two transistor circuit is shown in Figure 3.10. Again the high power silicon type was chosen so that the reader may compare the operation with that of Figure 3.6. Because of the large base-to-emitter potential of silicon transistors, R_2 and R_3 could be connected

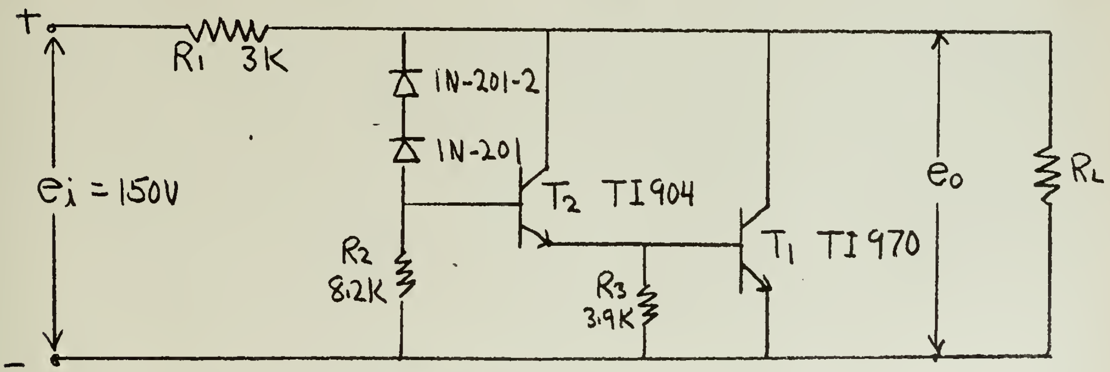

Figure 3.10 Example of Two Transistor Shunt Regulator

directly to ground in order to provide bias current for the diode and T_2. The operation of the circuit was even slightly better this way

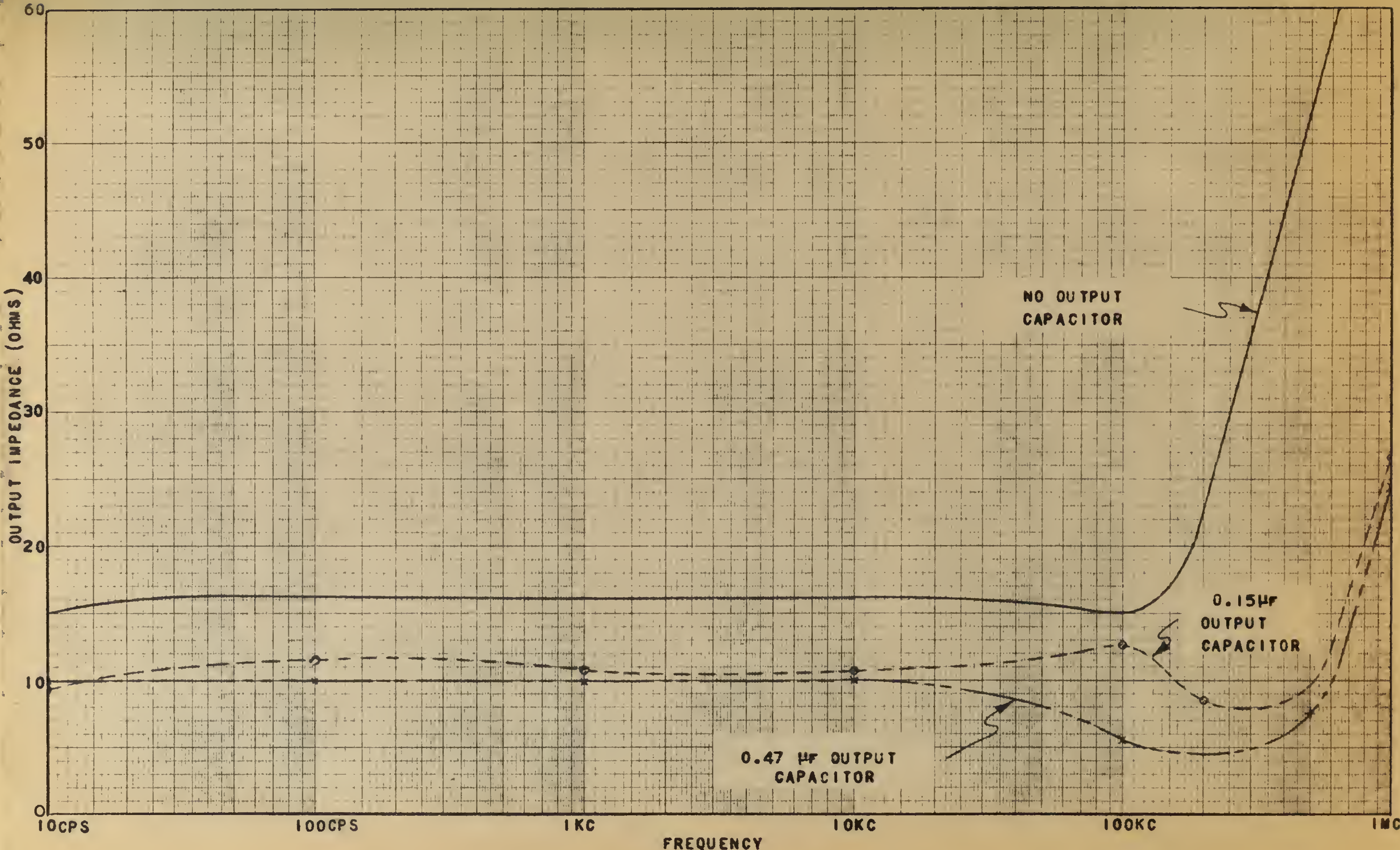

FIGURE 3.11 OUTPUT IMPEDANCE VARIATIONS WITH FREQUENCY FOR CIRCUIT OF FIGURE 3.10

than it was when a battery and larger resistors were used. The same technique has been successfully used with germanium transistors at low temperatures. The resistance values are much lower of course. At higher temperatures an external bias supply must be used for germanium units for the reasons discussed in Chapter II.

The performance of this circuit at both room and elevated temperatures was checked. The DC output resistance was about seven ohms, regulation was .00175 and the voltage change with a 77° C rise in temperature was .084 volts. Each of these tests show slight improvement over the circuit of Figure 3.6. Plots of output impedance versus frequency with no output capacitor and with two values of output capacitance are shown in Figure 3.11. This circuit can be seen to have quite reduced performance even at low frequencies unless a rather large output capacitor is used.

Several circuits that provide a means for varying the output voltage have been investigated. As was the case with the series regulator we must pay the price of reduced performance to gain this flexibility. The basic circuit of Figure 3.12 was suggested in Reference (13). The price paid for trimming in this instance is one transistor and slightly poorer

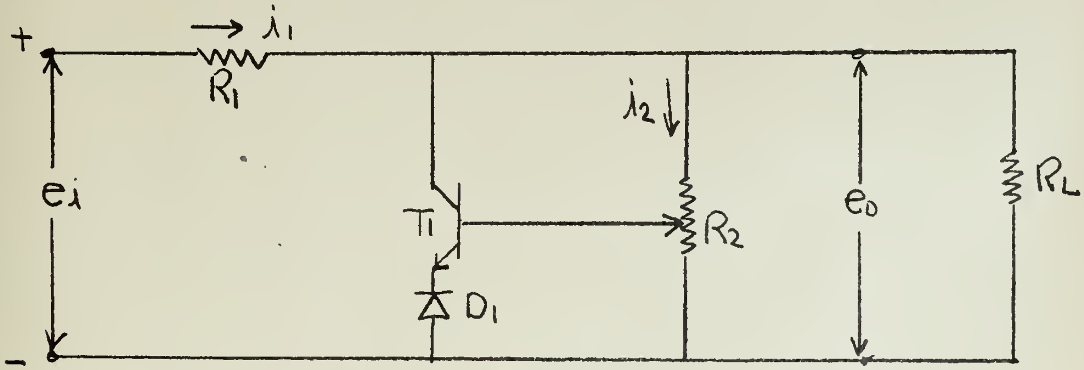

Figure 3.12 Basic Shunt Trimming Circuit

performance than could be obtained by the diode alone. The diode must absorb all load variations thus limiting operation to low power. In addition, the effective R_D is slightly greater than that of the diode unless R_2 is made quite small. This is because variations in i_b with different loadings reflect a change in i_2 and consequently V_{cb} and e_o. In addition, the circuit is not as flexible as it may first appear because of the limited range of permissable collector voltage. The resistor R_1 must either be small enough to provide current flow through T_1 at the full load current of the maximum voltage setting, or be made variable. Either alternative may be troublesome.

The circuit just discussed was not tested. An improved version shown in Figure 3.13 was constructed. The addition of the second transistor results in operation comparable to the one transistor shunt circuit of Figure 3.3. This is reasonable for we have reduced the current

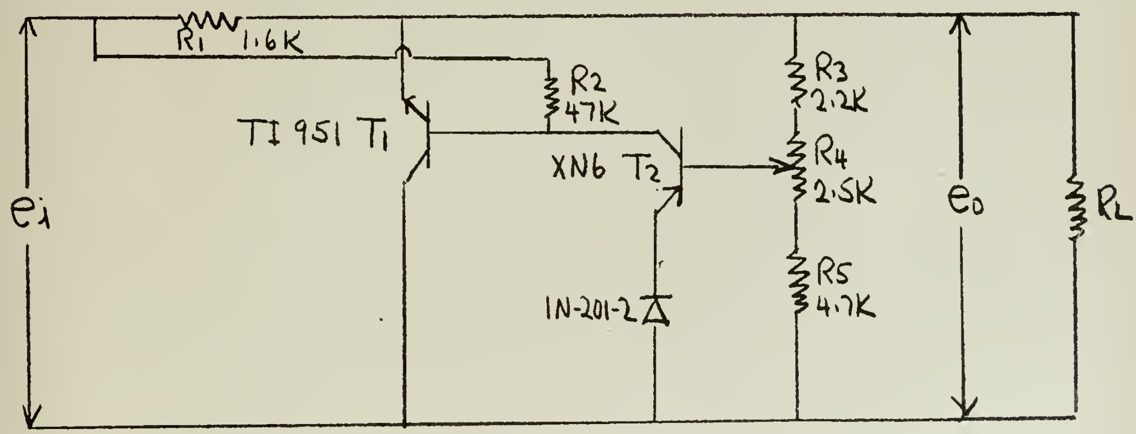

Figure 3.13 Trimming Circuit Employing Two Transistors

flow through the diode by the factor of $1/\beta_1$. Transistor T_2 is used only for isolating the voltage divider. Resistor R_2 is used to insure

that the diode and T_2 are operated at a satisfactory current level. The circuit was tested and found to have a voltage range from 13.2 to 19 volts. The regulation and output impedance obtained are shown in Table 3.1. As was the case with the series trimmer, the performance

e_o	Output Impedance	Regulation
-13.2V.	4.6 ohms	.005
-16 V.	5.6 ohms	.0067
-19 V.	7.15 ohms	.008

Table 3.1

decreases with output voltage because of the larger portion of R_4 associated with R_3. When the total resistance of the voltage divider was increased by a factor of ten, output impedance at 15 volts increased to 22.5 ohms and regulation was only .025. Again it is pointed out that the performance of this circuit is satisfactory, but we fail to utilize the full potentialities of the transistor.

The circuit shown in Figure 3.14 is a shunt circuit that was used to provide bias voltages for a radar modulator. It is included to indicate one practical application of these regulators. This circuit resulted in a considerable saving of space and improved performance over an existing circuit employing VR-tubes. In addition it eliminated the difficulty of VR-tube starting potential associated with light tight packaging. Both the -222 volt and -186 volt loads vary together but are of unequal magnitudes. The load currents range from zero to

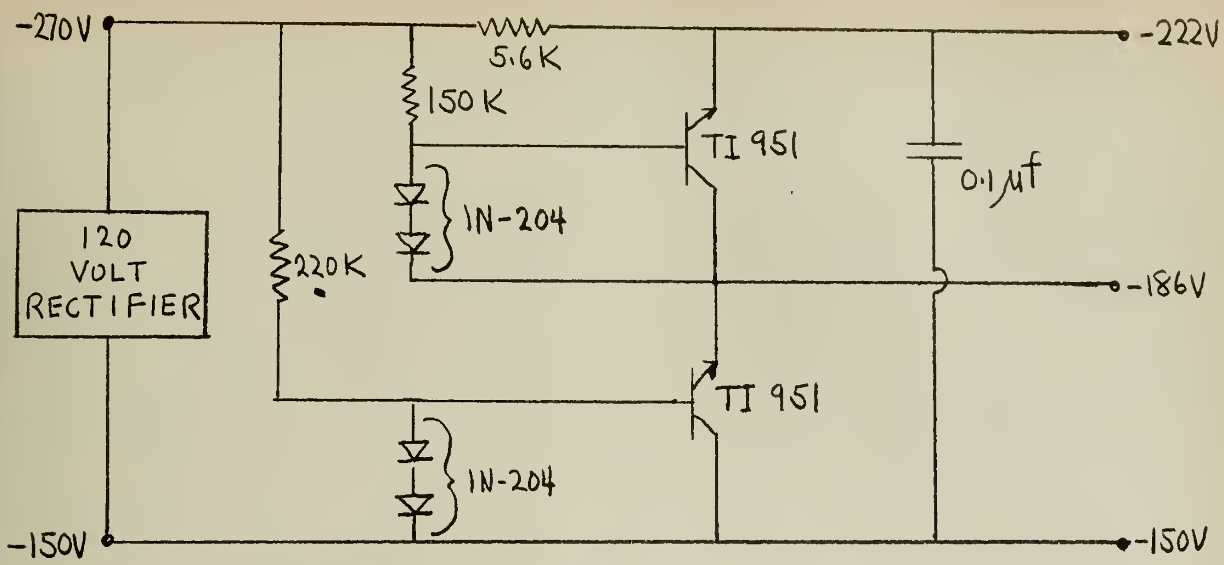

Figure 3.14 Example of Shunt Bias Supply

five ma. The output impedance of each load was checked for frequencies from DC to two megacycles. It remained in the range of 10 to 20 ohms dependent on load current. The output is quite stable with temperature. The change for each half was 1.5 volts over a temperature range of 100 degrees centigrade. Ripple was reduced a factor of 20.

The regulation and output impedance of a shunt regulator do not approach the optimum attainable by other methods. The power loss is appreciable. Nevertheless, the ease with which it can meet many specifications makes it well worth considering.

CHAPTER IV

THE EMITTER FOLLOWER VOLTAGE REGULATOR

Of the three basic transistor regulator types, the emitter follower displays the most dramatic improvement over its vacuum tube counterpart. This circuit, shown in its simplest form in Figure 4.1, has a gain that much more nearly approaches unity than does the cathode follower.

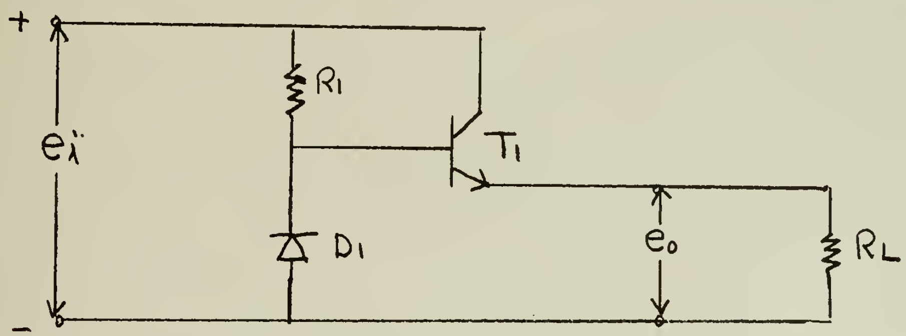

Figure 4.1 Basic Emitter Follower Voltage Regulator

Consequently its output impedance and regulation are quite superior to that of the cathode follower voltage regulator.

In addition to its good performance and simplicity, this configuration possesses other advantages. It may be designed with a low collector voltage thus permitting large load currents without exceeding the allowable dissipation of the transistor. Also, in every other regulator the author has encountered, the full output voltage is impressed across one or more transistors. This is not the case with the basic emitter follower or its modifications. It is therefore possible to operate this type of circuit at higher potentials without exceeding the maximum collector voltage rating of the transistors used. In other words, this configuration

generally permits the use of smaller transistors than do the other types.

The analysis of this circuit is made in terms of r, the voltage regulator output impedance factor as defined in Chapter II, and the overall regulation $\frac{e_o}{e_i}$. It was deemed adviseable to use this latter measure of effectiveness for regulation rather than $\mathcal{R}$, because the emitter resistance of the transistor cannot be separated from the load. The derivations are shown in Appendix VI and the results are

$$r \approx r_e + (1-\alpha)\left[r_b + \frac{R_1 R_D}{R_1 + R_D}\right] \tag{4.1}$$

and

$$\frac{e_o}{e_i} \approx \frac{R_L R_D}{(1-\alpha)\left[R_D(r_b + R_1) + r_b R_1\right] + (r_e + R_L)(R_1 + R_D)} . \tag{4.2}$$

If we make the assumptions that R_L is much greater than r_e, and R_1 is much greater than R_D, it can be shown that equation (4.2) is essentially

$$\frac{e_o}{e_i} \approx \frac{R_D}{R_1} . \tag{4.3}$$

A power transistor is usually used in this configuration. If the typical values enumerated in Chapter III are substituted in equation (4.1), the resultant output impedance is about seven ohms. Equation (4.3) indicates the basic shortcomings of the emitter follower circuit. Its regulation is dependent almost entirely upon the ratio of R_D to R_1. It is necessary to keep R_1 rather small, one to ten kilohms, so that the diode current will be ample at full load. Nevertheless, this means regulation from .01 to .001 if an R_D of 100 ohms is assumed.

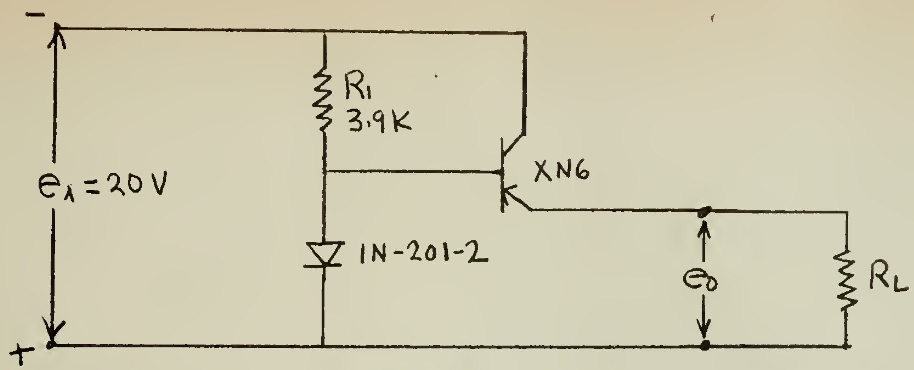

Figure 4.2 Typical Emitter Follower Voltage Regulator

The circuit shown in Figure 4.2 was constructed and tested. With an R_1 of 3.9 K the diode current was still 0.3 ma at full load when the input voltage was reduced to 15 volts. This insured satisfactory operation under the most adverse conditions expected. The DC output impedance of this circuit was measured to be 5.26 ohms and the regulation was .0057. The DC output impedance of the diode was measured to be 19 ohms. Using this value with the R_1 of 3.9 K in equation (4.3), the regulation computed is .0049. This demonstrates the worth of equation (4.3) for prediction of results.

The emitter follower regulator is one of the class whose value of k changes with load. In fact, it varies in the same manner as the symmetrical series regulator discussed in Chapter II. As was the case before, satisfactory predictions can be obtained by omitting the current loss factor.

If it is desirable to improve upon the regulation of the basic circuit it can be done quite easily by the addition of another transistor and diode as illustrated in Figure 4.3. The philosophy is the same that was used to improve the series regulator in Figure 2.9. Transistor T_2

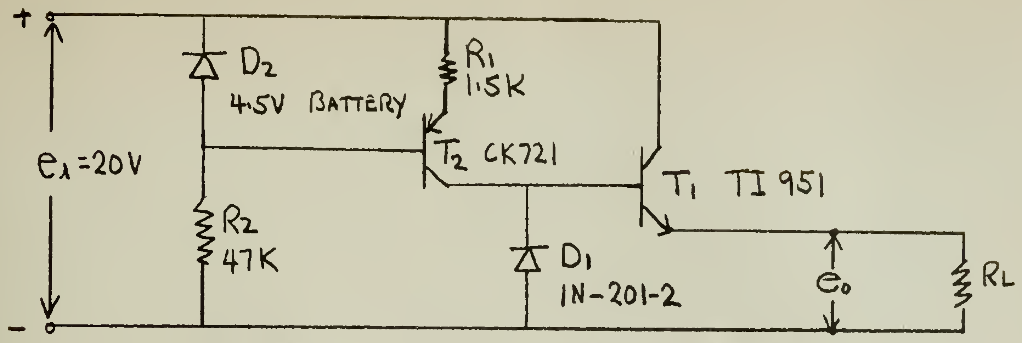

Figure 4.3 Method to Improve Regulation

is added as a constant current source for the diode D_1. The resistor R_1 is chosen so that T_2 is operated on the constant current portion of its characteristic curve. As the voltage across R_1 is held constant by D_2, input voltage variations appear only as collector voltage changes on T_2. Therefore i_{e_2} and the diode current remain essentially constant.

The circuit of Figure 4.3 was tested using component values indicated. The regulation is now .0009. The DC output impedance was measured as 7.6 ohms. The increase in r over the value obtained in the previous circuit can be attributed to the use of the TI 951 in place of the XN6. This configuration should not noticeably alter the output impedance. The output impedance remains quite stable with frequency change. Figure 4.4 indicates the results of tests with and without a stabilizing output capacitor.

A compensation technique may be employed if it is deemed desirable to reduce the output impedance. Figure 4.5 indicates a technique for accomplishing this. As the load current is increased, e_1 is raised due to the drop across R_3. This then increases i_{e_2} and the reference diode current. If R_3 is chosen properly, this increase in diode current with

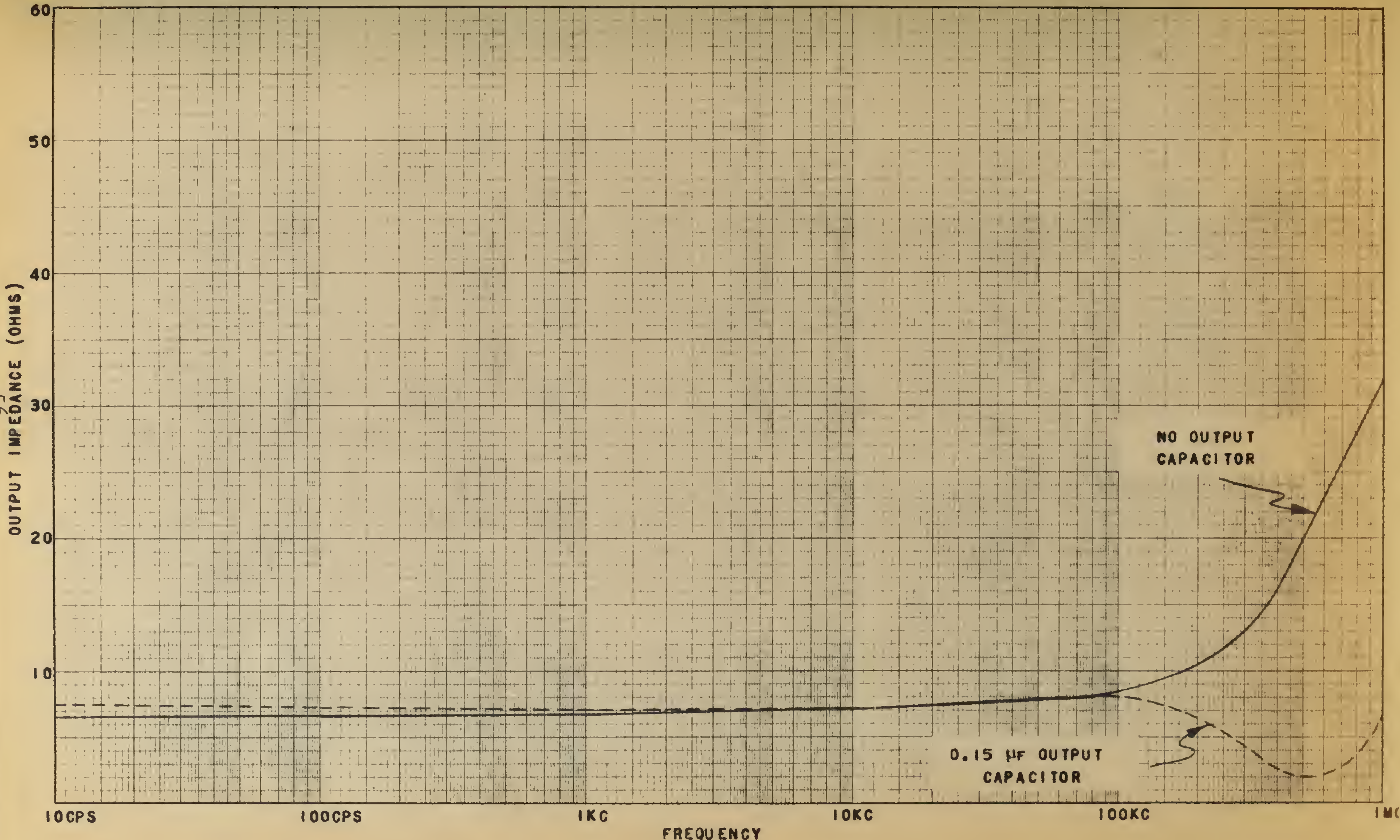

FIGURE 4.4 OUTPUT IMPEDANCE VARIATIONS WITH FREQUENCY FOR THE CIRCUIT OF FIGURE 4.3

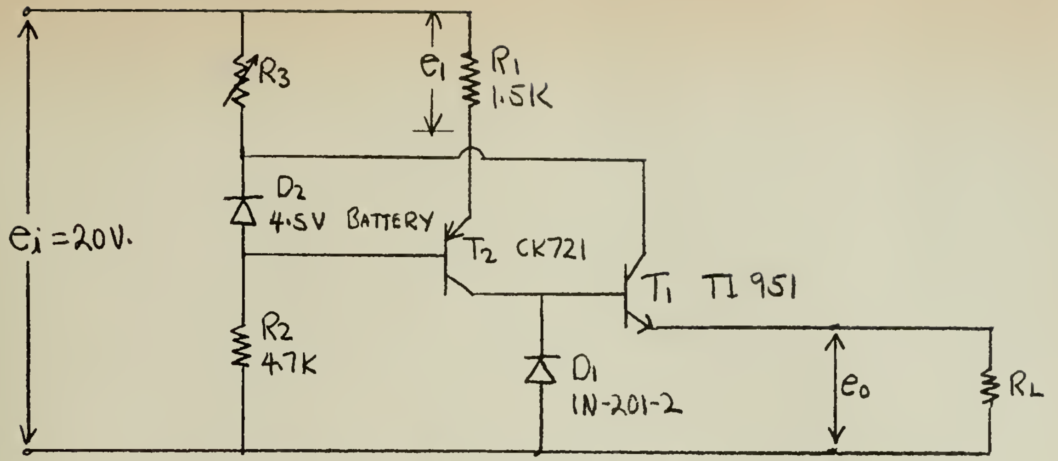

Figure 4.5 Method to Improve Output Impedance

load current can completely compensate for output voltage changes due to load variations. By physical reasoning we can determine the proper size for R_3 for zero output impedance. Equating voltage drops caused by the load current, i_o, we have

$$i_o r = \frac{R_D}{R_1}(i_o R_3)$$

Solving for R_3 we have

$$R_3 = \frac{R_1}{R_D}(r) \tag{4.4}$$

Substituting the values of

$R_1 = 1.5$ K

$R = 19$ ohms

$r = 7.6$ ohms,

the necessary R_3 for perfect compensation is 600 ohms. However, if 600 ohms were used in this circuit, excessive diode currents would be attained at full load. At 24 ma the drop across R_3 would be in the order of 15 volts which would increase i_{e2} and current through diode D_1 to a

value that could damage these units. To demonstrate the validity of the reasoning a 100 ohm resistor was added in series with D_1 thus raising its effective R_D to 119 ohms. The value of R_3 was calculated to be 96 ohms. A potentiometer was used for R_3 and its value for zero output impedance was measured to be 120 ohms.

The emitter follower voltage regulator is a very useful device. It shares some of the advantages of both the shunt and series regulators. In its basic form, it is comparable in simplicity to the shunt regulator. Their performance is quite similar. The choice between the two rests on the circuit application and the nature of the available driving voltage source. The two transistor emitter follower regulator is superior to the basic symmetrical series regulator for regulation. Its output impedance is not as low unless compensation is used. The three-transistor series regulator, however, is capable of superior performance for both regulation and output impedance.

CHAPTER V

THE SILICON JUNCTION DIODE AS A VOLTAGE REFERENCE ELEMENT

The current-voltage characteristics of a typical silicon junction diode are illustrated in Figure 5.1. For both forward and reverse voltages there are regions where the voltage is almost independent of current. These two saturation regions are separated by a region of very high back resistance. At a temperature of 25° C this back resistance is in the order of ten megohms at minus one volt. There are many circuit applications where the characteristics of this diode may be used to advantage. However, our interest is in the usefulness of this device as a voltage reference element so attention will be directed to the two saturation regions.

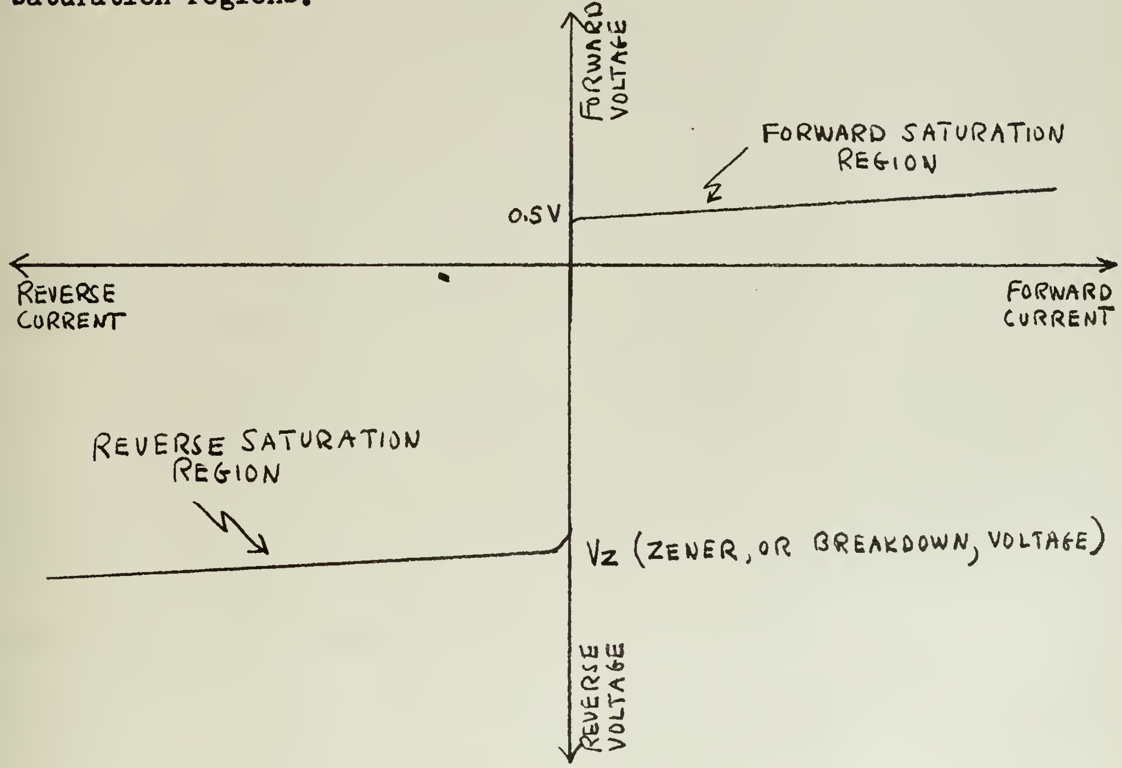

Figure 5.1 Characteristics of a Typical Silicon Junction Diode

The reverse voltage where the resistance suddenly decreases from megohms to a value often less than 100 ohms is referred to by several names. Zener voltage, reverse breakdown voltage, reference voltage and reverse saturation voltage are the most common. Figure 5.2 illustrates this phenomena by an expanded plot of the characteristics of a Texas Instrument 653C9. This diode is considered typical of commercially available low breakdown voltage units. By varying construction techniques values of the breakdown potential may be obtained over a range from a few volts to several hundred. This is slightly misleading. It will be shown later that the features desirable for a reference standard are degraded as the breakdown potential is increased. The reader interested in the physical explanation for this breakdown phenomena is referred to the literature[15][16]. Reference (17) evaluates the worth of the junction diode for other applications and also discusses the manufacturing process.

The voltage breakdown in the reverse direction is the one most useful for reference purposes. The forward saturation region does possess some features that can often aid the designer of voltage regulators. An expanded plot of the forward characteristics of the 653C9 is shown in Figure 5.3. One unique feature is that this portion of the characteristic is nearly the same for all silicon alloy junction diodes regardless of reverse breakdown potential. Numerous different diodes possessing widely different breakdown potentials, and made by several manufacturers, were tested. The separation in voltage at like current values never exceeded 0.15 volts. The slope, or dynamic resistance, of all units was on the order of 50 ohms in the one to three milliampere

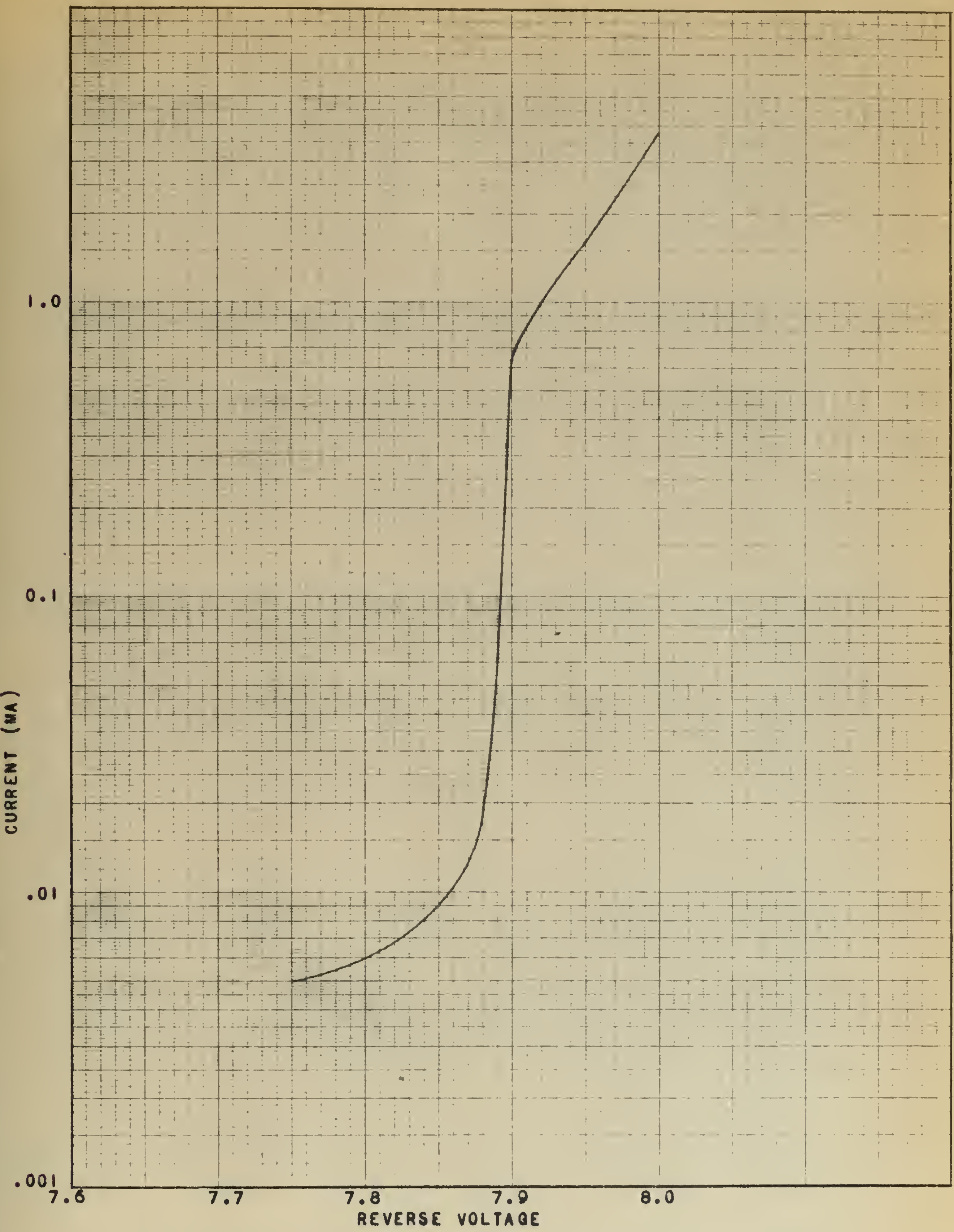

FIGURE 5.2 EXPANDED PLOT OF THE REVERSE REGION OF A 653C9

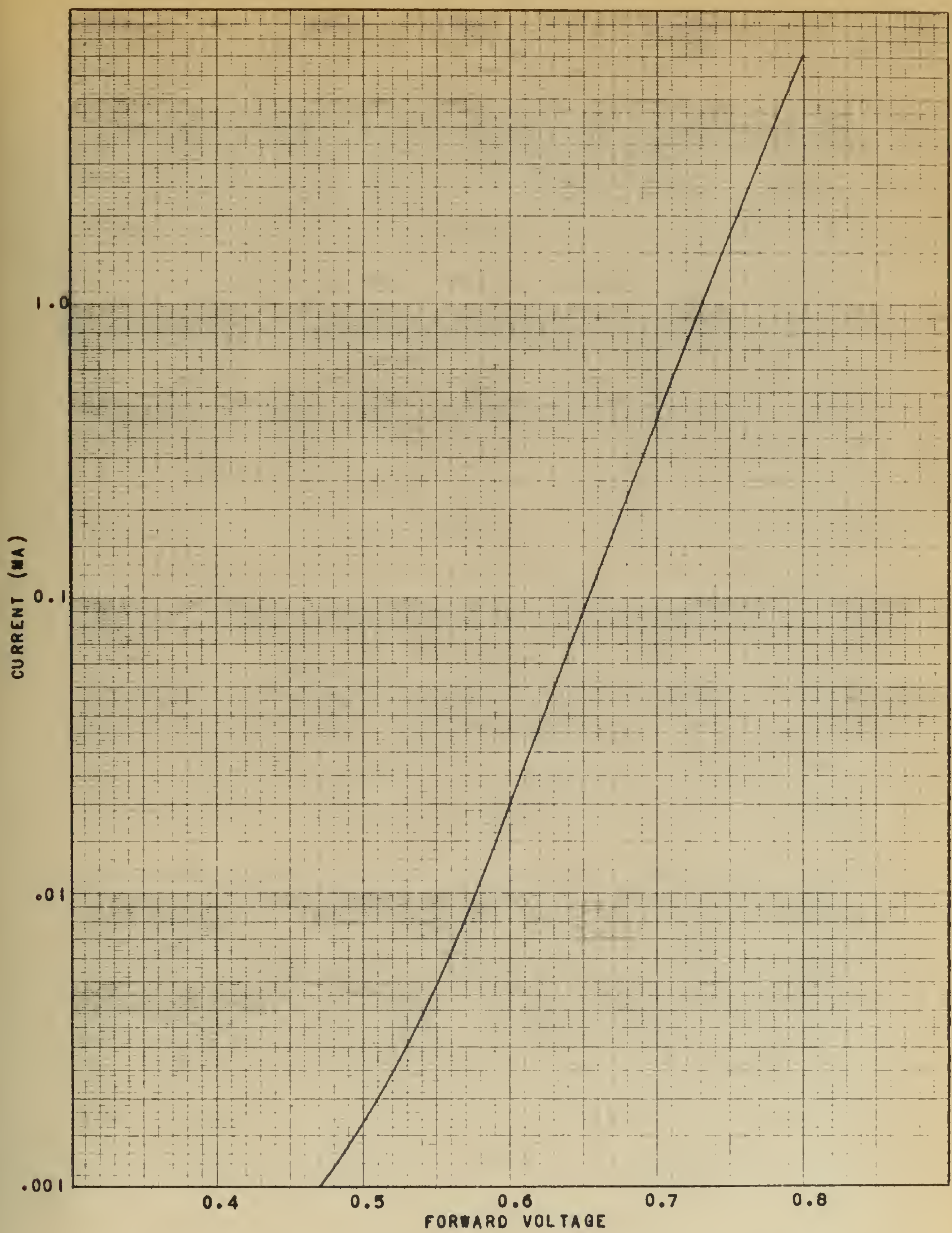

FIGURE 5.3 EXPANDED PLOT OF THE FORWARD REGION OF THE 653C9

region. This is comparable to the dynamic resistance of the reverse saturation region.

In order to evaluate the worth of a device as a voltage reference element a number of its features should be considered. Among the most important are: (1) reference voltages available; (2) dynamic resistance; (3) permissable power dissipation; (4) variations with temperature; (5) starting potential; (6) noise and other random variations; (7) ruggedness. The merits of the reverse voltage breakdown region of the silicon junction diode will be considered for each feature.

The continuous range of reference voltages has already been mentioned. However, other features are dependent upon the breakdown potential value. Figure 5.4 has been reproduced from Reference (5) with the kind permission of the publisher. Nomenclature has been altered to agree with that used in this paper. It illustrates the dependence of the temperature coefficient, C, and the dynamic resistance, R_D, with breakdown, or reference potential, V_z. The slope of the voltage-current characteristic is termed the dynamic resistance. The temperature coefficient is defined as the percent change in reference voltage per degree Centigrade. This is an empirical curve based upon Smith's own tests. Nevertheless the units tested, regardless of manufacturer, have fit this curve fairly closely. Smith also points out that C is constant over a wide range of current and that R_D is constant over a wide range of temperatures. It can be seen that the desirable features of low R_D and C are rapidly lost as V_z is increased. In addition the ease in which diodes may be manufactured to a specific value of V_z decreases with increasing V_z. One manufacturer states that in order to obtain

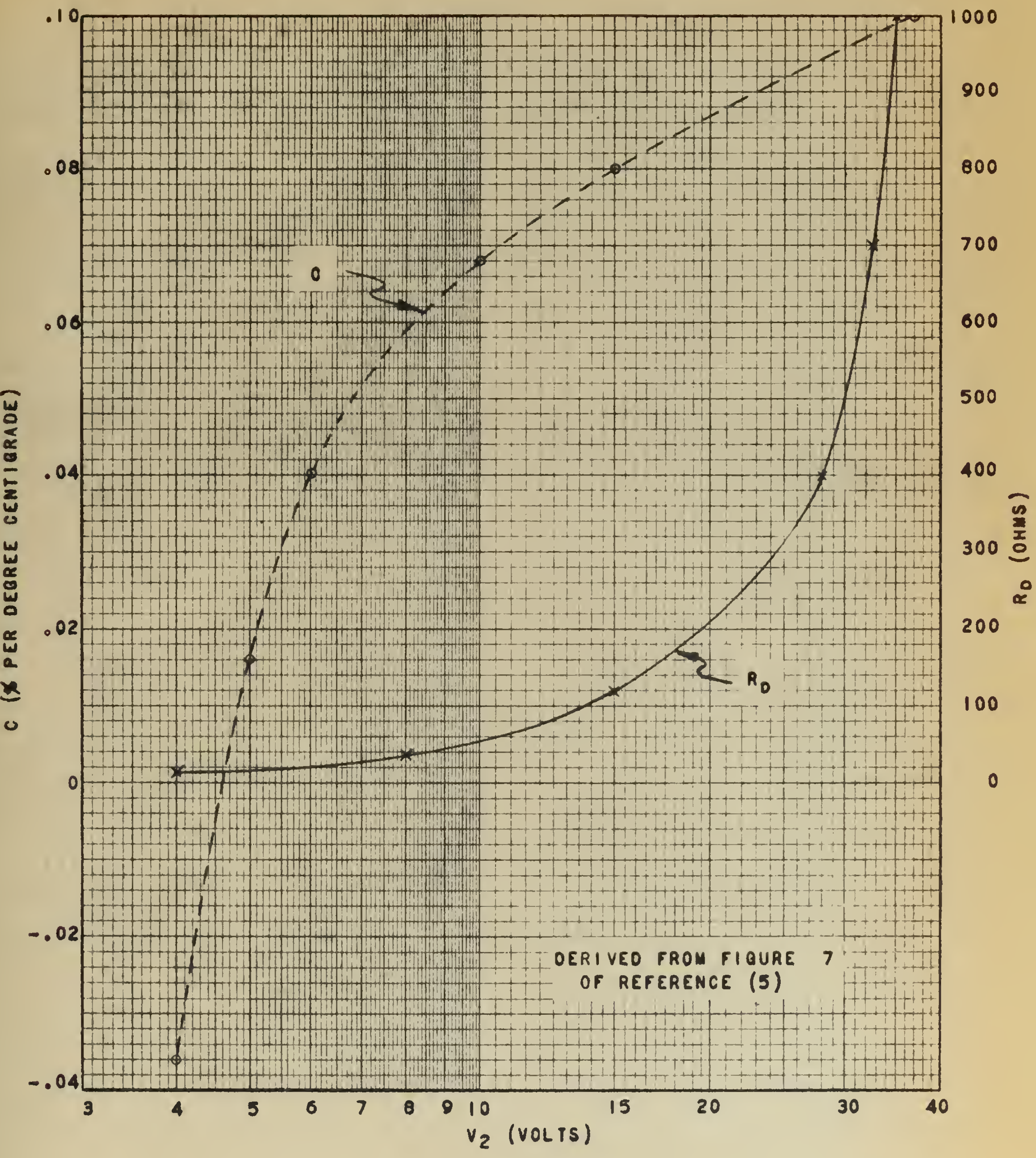

FIGURE 5.4 R_D AND C VERSUS V_Z FOR TYPICAL SILICON JUNCTION DIODES

satisfactory reference diodes to within five percent accuracy of a specific V_z greater than ten volts, about eighty-five percent rejects can be expected. This construction difficulty, and the general poor performance at high V_z, has led most semi-conductor manufacturers to advertise only the diodes with a reference potential less than ten volts as "reference diodes". These can be obtained in a number of specific values and with breakdown potential to within one to five percent of the nominal value at 25° C. The diodes with higher breakdown voltages are usually grouped into types by specifying V_z within a considerable voltage range. For example, this could be stated as "from 12 to 18 volts", or V_z may be specified to be greater than a certain voltage. These diodes are intended as switches and are not recommended as precise voltage standards unless considerable selection of individual units is exercised.

The maximum allowable power dissipation listed for these diodes vary somewhat from manufacturer to manufacturer. The majority fall in the range from 125 to 150 milliwatts at 25° C. These are usually derated one milliwatt per degree Centigrade for temperatures above 25° C. This power rating sometimes necessitates the use of extra transistors in voltage regulator circuits merely to protect the diode.

The question of starting potential, which is often a severe problem when VR-tubes are used, is eliminated by silicon junction diodes. The starting and working potentials of these diodes are almost identical. The ever increasing practice of packaging commercial and military equipment in light tight boxes, thus aggravating the starting potential difficulties with VR-tubes, often makes this feature of the diode quite attractive.

The silicon junction diode evinces none of the long time instability of VR-tubes. Reference (3) may be consulted for a discussion of the variations exhibited by VR-tubes. The diode does exhibit some noise type variations. The noisy region is limited to currents in the vicinity of the "Zener knee". One manufacturer states that the amplitude of these noise peaks may reach peak values of three volts rms at currents below 200 microamperes. Pearson[15] states that this noise is not inherent to the device but is dependent upon the manufacturing technique used. The more recent manufacturers data sheets make no mention of this noise. Nevertheless, it is usually a simple enough matter to design the regulator circuit so that diode current is maintained above the noisy region.

These units are extremely rugged. One manufacturer has published the results of quite extensive tests.[17] These diodes underwent: (1) Vibration tests from ten to five hundred cycles per second which included accelerations to fifty G; (2) Acceleration tests of 125 G; (3) Impact shock tests to 145 G; (4) Altitude tests to 100,000 feet; (5) Extensive moisture resistance and humidity checks. Neither erratic electrical behavior nor mechanical failure was observed during these tests.

The apparent shortcomings of this device are the degradation of dynamic resistance and temperature coefficient at the higher breakdown potential, and the low allowable power dissipation, especially at elevated temperature. The effects of these features may often be minimized by the use of several diodes in series. Figure 5.4 indicates that the variation of R_D and C with V_z is far from linear. By the substitution of three, eight volt diodes in series to obtain a 24 volt

reference, the value of R_D is about 60 ohms instead of 300. The change in voltage for a hundred degree Centigrade temperature rise is reduced from 2.3 volts to 1.5 volts. In addition, the extra diodes will triple the allowable power dissipation. This technique has been used extensively by the author in designing voltage regulators required to operate over the temperature range from -55° C to 125° C.

If one is willing to expend the effort it is possible to tailor the temperature characteristics to any desired value by the addition of diodes operated in the forward direction. The forward temperature characteristics of several diode types are shown in Figure 5.5. These indicate that the forward temperature coefficient possesses a sign opposite to that of the reverse region. It was noted earlier that the forward R_D is also quite low. A combination of forward and reverse connected diodes could conceivably be made to give almost any desired V_z and a zero temperature coefficient. This is not very practical unless the desired V_z is small. The forward temperature coefficient is fairly large, but its breakdown voltage is low. Therefore, the absolute voltage change for each diode is not very great. Fortunately most transistor-junction diode voltage regulator configurations are such that the internal diodes of the transistors compensate for the temperature variations of the junction reference diode in a similar manner.

In Figure 5.6 a symmetrical series voltage regulator possessing extremely stable temperature characteristics is shown. This circuit is essentially the same as the one shown in Figure 2.7 except for the addition of the two forward biased diodes. The silicon transistors were substituted for the XN6 and CK 721 to permit tests at elevated

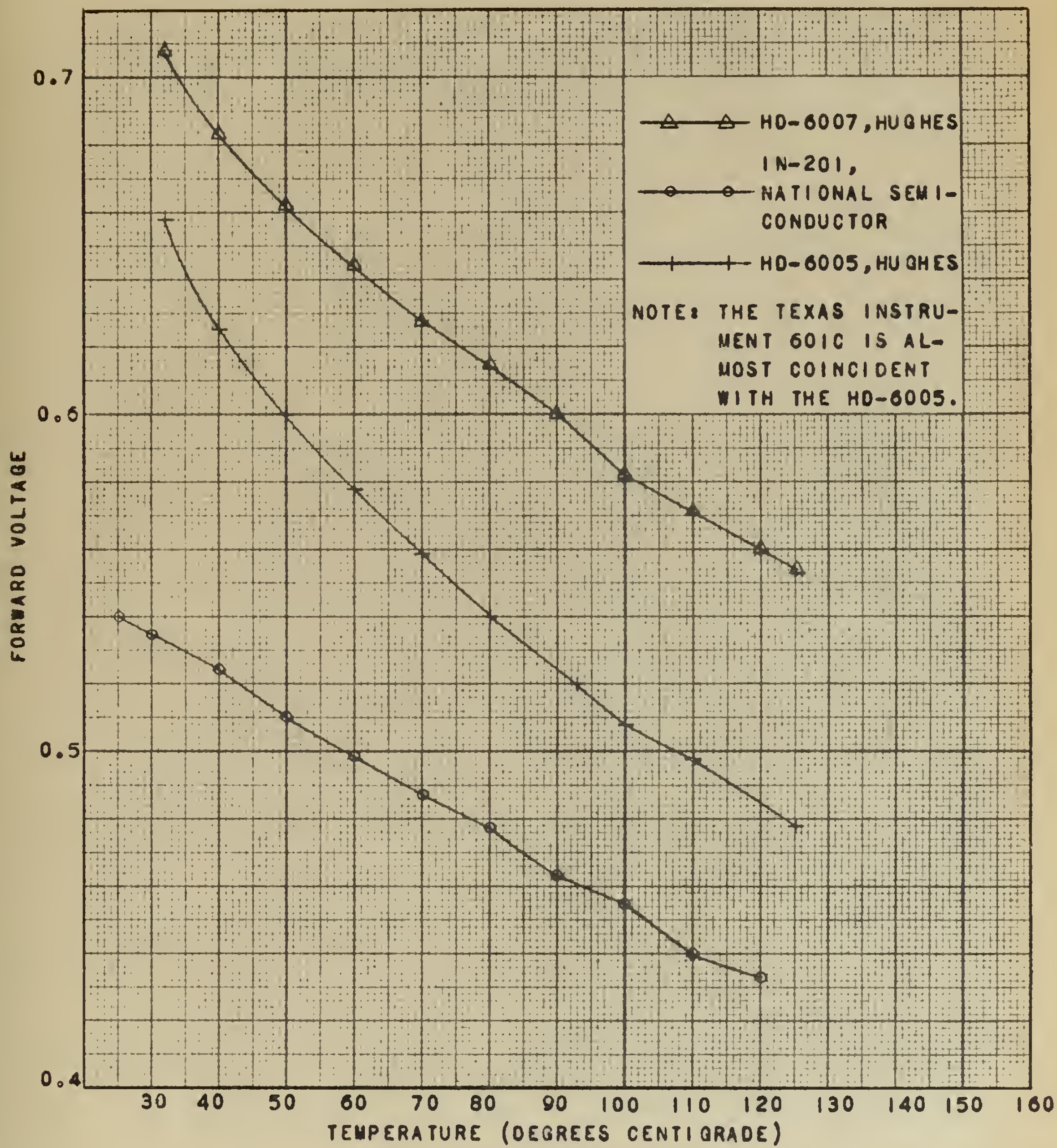

FIGURE 5.5 FORWARD TEMPERATURE CHARACTERISTICS OF SEVERAL SILICON JUNCTION DIODES

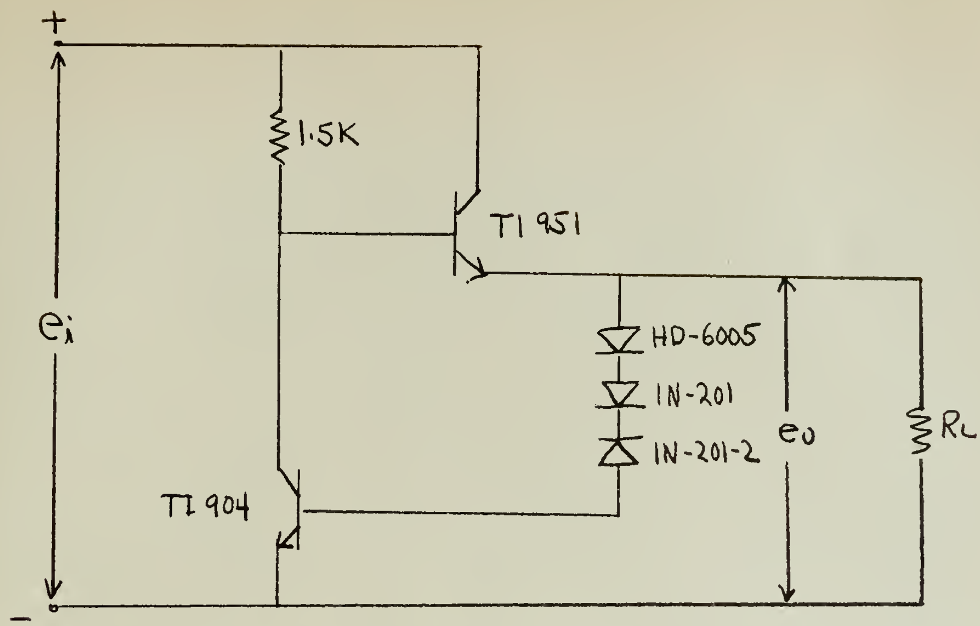

Figure 5.6 Example of Voltage Regulator With a Near Zero Temperature Coefficient

temperatures. The variation in output voltage with temperature is shown in Figure 5.7. For comparison purposes, the results of a test with the HD 6005 and IN-201 omitted are included. The output voltage at 25° C was 11.9 volts for the three diode circuit and 10.7 volts for the uncompensated arrangement. This illustrates the feasibility of using forward biased diodes to stabilize a voltage regulator against temperature variations.

The silicon junction diode is an excellent reference element at low voltages. Its dynamic resistance and other features are far superior to those of other references such as batteries and VR-tubes. In the intermediate voltage range, ten to sixty volts, its desirable features begin to fall off, but with proper circuit design these diodes may be used with satisfactory results. In the ranges where the VR-tube operates,

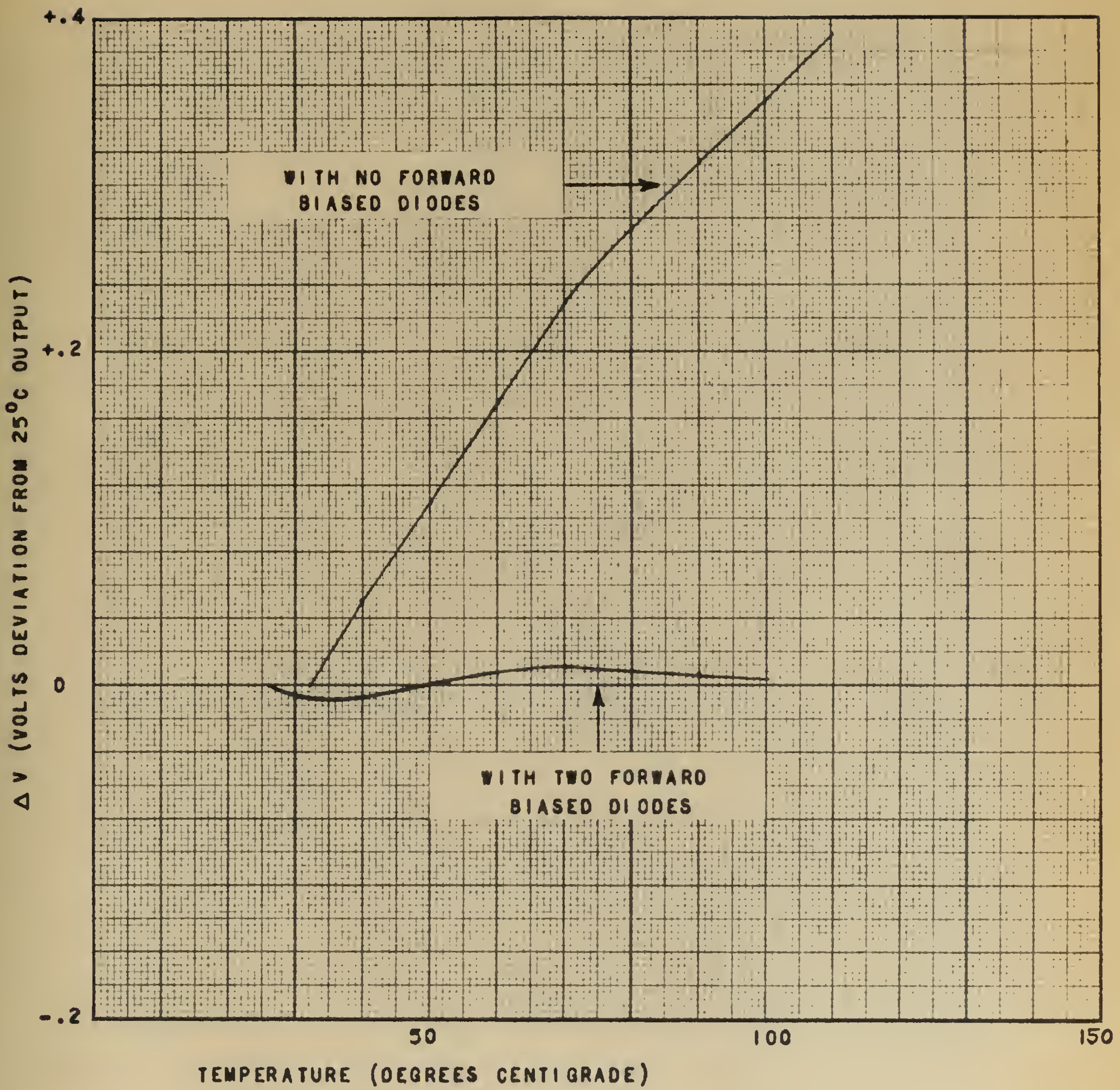

FIGURE 5.7 TEMPERATURE CHARACTERISTICS OF THE CIRCUIT OF FIGURE 5.6

sixty volts and above, the diode must wait for advances in the state of the art before it can compete with the VR-tube.

CHAPTER VI

CONCLUSIONS

This paper has presented three basic types of circuits employing transistors and junction diodes that are suitable for providing a stable source of DC voltage. Modifications to improve or alter the performance of each original form have been discussed. Particular advantages and the applications most suitable for each have been covered in the body of the thesis.

It has been demonstrated that a transistor-junction diode voltage regulator can be designed to meet almost any conceivable specifications within the power handling capacities of the transistors. In order to take advantage of the peculiar characteristics of a transistor it is recommended that provisions for varying the output voltage be excluded whenever possible. If it is desired to hold the nominal output voltage to a close tolerance this should be done by close selection of the reference diodes.

The various regulators presented were analyzed for low frequencies by circuit theory and experimentation. The high frequency performance was investigated by experimental methods only. It is suggested that further research should include the analysis of these circuits using the high frequency equivalent circuit of the junction transistor.

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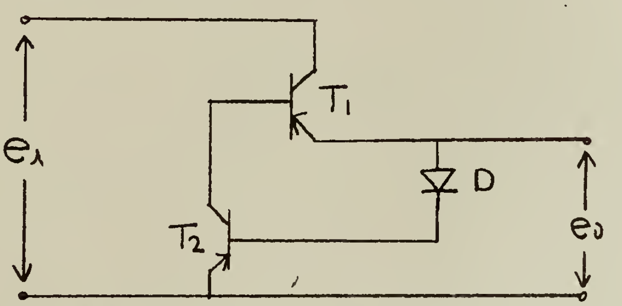
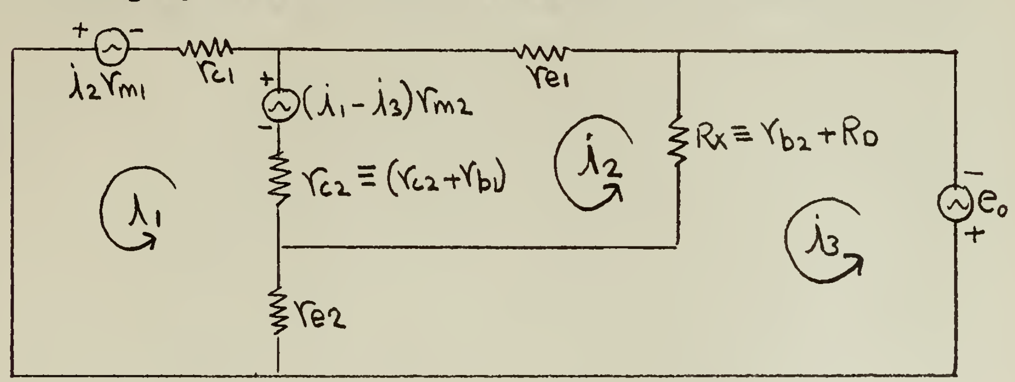
APPENDIX I

DERIVATIONS FOR THE ULTIMATE SYMMETRICAL SERIES VOLTAGE REGULATOR

The circuit for the ultimate symmetrical series voltage regulator of Figure 2.6 is repeated below.



The following equivalent circuit is used for the derivation of r.



Writing the mesh equations we obtain the system determinant:

$$\Delta = \begin{vmatrix} (r_{c1} + r_{c2} + r_{e2} - r_{m2}) & -(r_{c2} + r_{m1}) & -(r_{e2} - r_{m2}) \\ -(r_{c2} - r_{m2}) & (r_{e1} + r_{c2} + R_x) & -(R_x + r_{m2}) \\ -(r_{e2}) & -(R_x) & (r_{e2} + R_x) \end{vmatrix} .$$

Adding the second and third columns to the first, and then adding the first and third rows of the resulting determinant to the second, we can manipulate Δ into the form .

$$\Delta = \begin{vmatrix} (r_{c1} - r_{m1}) & -(r_{c2} + r_{m1}) & -(r_{e2} - r_{m2}) \\ (r_{c1} - r_{m1} + r_{e1}) & (r_{e1} - r_{m1}) & 0 \\ 0 & -(R_x) & (r_{e2} + R_x) \end{vmatrix} .$$

Solving the determinant we have

$$\Delta = (r_{c1} - r_{m1})(r_{e1} - r_{m1})(r_{e2} + R_x) + R_x(r_{e2} - r_{m2})(r_{c1} - r_{m1} + r_{e1})$$
$$+ (r_{c2} + r_{m1})(r_{e2} + R_x)(r_{c1} - r_{m1} + r_{e1}) .$$

Noting that r_e is much less than r_m and using the identity

$$r_c = r_m$$

we have

$$\Delta \approx (r_{c2} + \alpha_1 r_{c1})(r_{e2} + R_x)(r_{c1})(1 - \alpha_1) - \alpha_1 r_{c1}^2 (1 - \alpha_1)(r_{e2} + R_x)$$
$$- \alpha_2 (1 - \alpha_1) r_{c1} r_{c2} R_x .$$

This may be manipulated into the more convenient form

$$\Delta \approx r_{c1} r_{c2} (1 - \alpha_1)\left[r_{e2} + (1 - \alpha_2) R_x\right] .$$

In order to solve for i_3 the following determinant may be written:

$$i_3 = \frac{1}{\Delta} \begin{vmatrix} (r_{c1} + r_{c2} + r_{e2} - r_{m2}) & -(r_{c2} + r_{m1}) & 0 \\ -(r_{c2} - r_{m2}) & (r_{e1} + r_{c2} + R_x) & 0 \\ -(r_{e2}) & -(R_x) & -e_0 \end{vmatrix} .$$

Solving the determinant results in

$$i_3 = \frac{-e_0}{\Delta}\left[(r_{e1} + r_{c2} + R_x)(r_{c1} + r_{c2} + r_{e1} - r_{m2}) - (r_{c2} - r_{m2})(r_{c2} + r_{m1})\right] .$$

This may be put in a more convenient form by neglecting r_{e1} and R_x in connection with r_{c2}, and by using the same approximation made above. The result is

$$i_3 \approx \frac{-e_0 r_{c1} r_{c2}}{\Delta}\left[1 - \alpha_1(1-\alpha_2)\right].$$

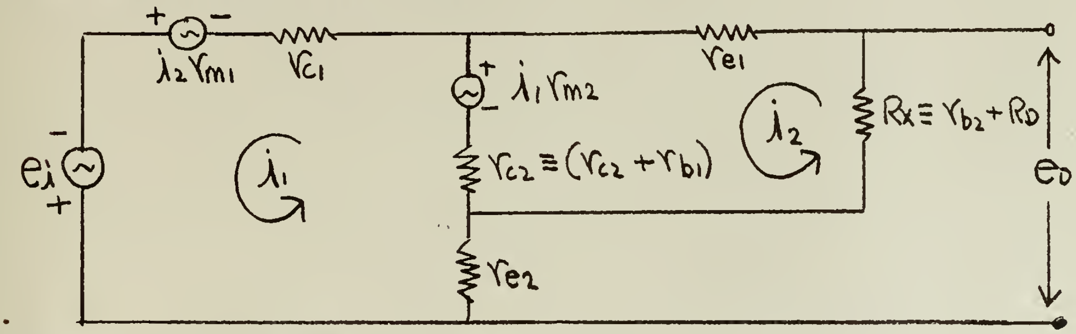
Now

$$r = -\frac{\partial e_0}{\partial i_0} = -\frac{e_0}{i_3}.$$

By algebraic manipulation we obtain the final result

$$r \approx \frac{(1-\alpha_1)\left[r_{e2} + (1-\alpha_2)(r_{b2} + R_D)\right]}{1 - \alpha_1(1-\alpha_2)}.$$

In order to derive the expression for $\mathcal{R}$ it is necessary to draw another equivalent circuit.



The system determinant for this circuit is

$$\Delta = \begin{vmatrix} (r_{c1} + r_{c2} + r_{e2} - r_{m2}) & -(r_{c2} + r_{m1} \\ -(r_{c2} - r_{m2}) & (r_{c2} + r_{e1} + R_x) \end{vmatrix}.$$

Solving this determinant and making the same approximations used in the derivation for r we have

$$\Delta \approx r_{c1} r_{c2}\left[1 - \alpha_1(1-\alpha_2)\right].$$

The expression for i_1 is

$$i_1 = \frac{e_i}{\Delta}\left(r_{c2} + r_{e1} + R_x\right)$$

while i_2 equals

$$i_2 = \frac{e_i}{\Delta} r_{c2}(1 - \alpha_2).$$

The output voltage can now be solved for.

$$e_o = i_1 r_{e2} + i_2 R_x = \frac{e_i}{\Delta}\left[r_{e2}\left(r_{c2} + r_{e1} + R_x\right) + r_{c2} R_x (1 - \alpha_2)\right]$$

The voltage regulator regulation factor is

$$\mathcal{R} = \frac{\partial e_o}{\partial e_i} = \frac{e_o}{e_i}$$

$$\mathcal{R} = \frac{r_{e2}\left(r_{c2} + r_{e1} + R_x\right) + r_{c2} R_x (1 - \alpha_2)}{r_{c1} r_{c2}\left[1 - \alpha_1(1 - \alpha_2)\right]}.$$

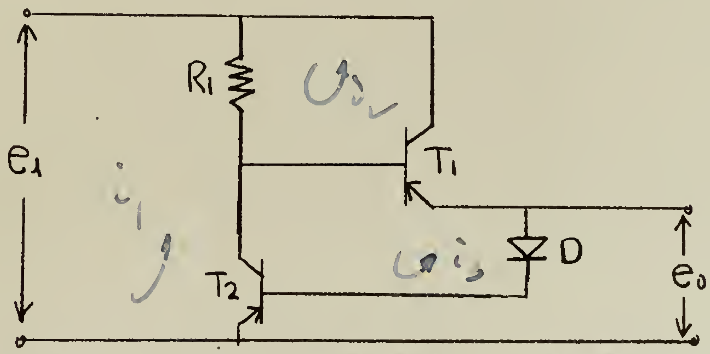
As the sum of r_{e1} and R_x is much less than r_{c2}

$$\mathcal{R} \approx \frac{r_{e2} + (1 - \alpha_2)(r_{b2} + R_D)}{r_{c1}\left[1 - \alpha_1(1 - \alpha_2)\right]}.$$

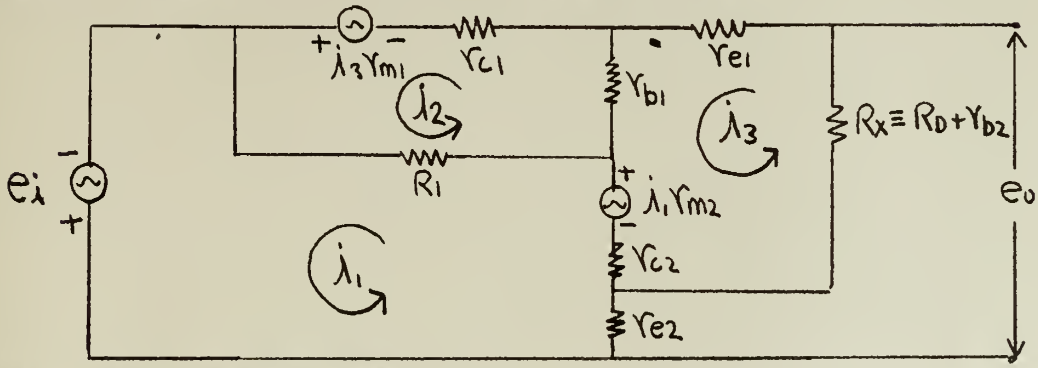
APPENDIX II

DERIVATION OF VOLTAGE REGULATOR REGULATION FACTOR FOR PRACTICAL SYMMETRICAL SERIES VOLTAGE REGULATOR

The circuit for the practical symmetrical series voltage regulator is shown below.



The equivalent circuit to be used for the derivation of the voltage regulator regulation factor, $\mathcal{R}$, is shown below.



The system determinant is

$$\Delta = \begin{vmatrix} (R_1 + r_{c2} - r_{m2} + r_{e2}) & -(R_1) & -(r_{c2}) \\ -(R_1) & (R_1 + r_{c1} + r_{b1}) & -(r_{b1} + r_{m1}) \\ -(r_{c2} - r_{m2}) & -(r_{b1}) & (r_{c2} + r_{b1} + R_x + r_{e1}) \end{vmatrix} .$$

By adding the second and third rows to the first the following somewhat simpler form is obtained.

$$\Delta = \begin{vmatrix} (r_{e2}) & (r_{c1}) & -(r_{m1} - R_x - r_{e1}) \\ -(R_1) & (R_1 + r_{c1} + r_{b1}) & -(r_{b1} + r_{m1}) \\ -(r_{c2} - r_{m2}) & -(r_{b1}) & (r_{c2} + r_{b1} + R_x + r_{e1}) \end{vmatrix} .$$

Solving the determinant we have

$$\Delta = r_{e2}(R_1 + r_{c1} + \underline{r_{b1}})(r_{c2} + \underline{r_{b1} + R_x + r_{e1}}) + r_{c1}(\underline{r_{b1}} + r_{m1})(r_{c2} - r_{m2})$$
$$- r_{b1}R_1(r_{m1} - \underline{R_x - r_{e1}}) - (r_{m1} - \underline{R_x - r_{e1}})(R_1 + r_{c1} + \underline{r_{b1}})(r_{c2} - r_{m2})$$
$$+ r_{c1}R_1(r_{c2} + \underline{r_{b1} + R_x + r_{e1}}) - r_{e2}r_{b1}(\underline{r_{b1}} + r_{m1}) .$$

The terms underlined in the expression above may be neglected. By algebraic manipulation Δ may be put in the form

$$\Delta \approx r_{c1}r_{c2}\left\{R_1\left[1 - \alpha_1(1-\alpha_2)\right] + r_{e2}\right\} - R_1\left[\alpha_1 r_{c1} r_{b1} - r_{e2} r_{c2}\right]$$
$$- \alpha_1 r_{c1} r_{b1} r_{e2} .$$

The last two terms are several orders of magnitude less than the first. The system determinant is to a good approximation

$$\Delta \approx r_{c1}r_{c2}\left\{R_1\left[1 - \alpha_1(1-\alpha_2)\right] + r_{e2}\right\} .$$

It is now necessary to solve for the currents i_1 and i_3. The exact and simplified expressions are shown below. Underlined terms are omitted in the simplified form.

$$i_1 = \frac{e_i}{\Delta}\left[(R_1 + r_{c1} + \underline{r_{b1}})(r_{c2} + \underline{r_{b1} + R_x + r_{e1}}) - r_{b1}(\underline{r_{b1}} + r_{m1})\right]$$

$$i_1 \approx \frac{e_i}{\Delta}\left[r_{c2}(R_1 + r_{c1}) - \underline{\alpha_1 r_{c1} r_{b1}}\right] \approx \frac{e_i}{\Delta} r_{c2}(R_1 + r_{c1})$$

$$i_3 = \frac{e_i}{\Delta}\left[(R_1 + r_{c1} + \underline{r_{b1}})(r_{c2} - r_{m2}) + \underline{R_1 r_{b1}}\right]$$

$$i_3 \approx \frac{e_i}{\Delta} r_{c2}(1-\alpha_2)(R_1 + r_{c1})$$

The voltage regulator regulation factor is

$$\mathcal{R} = \frac{e_0}{e_i} = \frac{i_1 r_{e2} + i_2 (R_0 + r_{b2})}{e_i} .$$

Substituting into the above the final form is obtained.

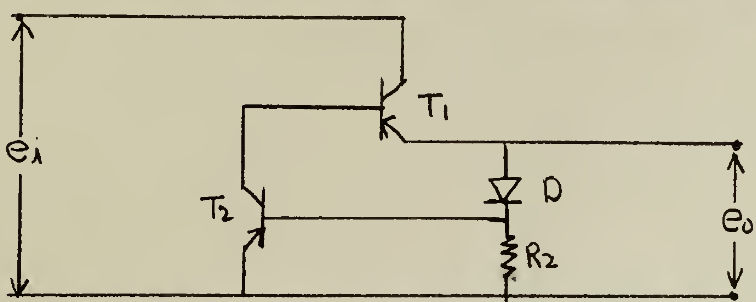
$$\mathcal{R} \approx \frac{(R_1 + r_{c1})\left[r_{e2} + (1-\alpha_2)(R_0 + r_{b2})\right]}{r_{c1}\left\{r_{e2} + R_1\left[1-\alpha_1(1-\alpha_2)\right]\right\}}$$

It can be shown that this expression reduces to the $\mathcal{R}$ of the ultimate symmetrical series regulator if R_1 is allowed to approach infinity.

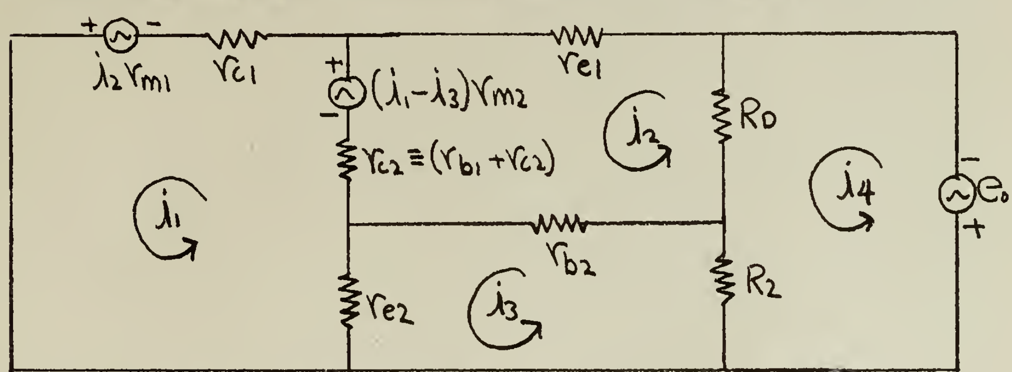
APPENDIX III

DERIVATION OF THE VOLTAGE REGULATOR OUTPUT IMPEDANCE FACTOR FOR PRACTICAL SYMMETRICAL SERIES VOLTAGE REGULATOR EMPLOYING A BIAS RESISTOR

The circuit for the ultimate symmetrical series voltage regulator modified to include a biasing resistor R_2 is repeated below.



The following equivalent circuit is used for the derivation of r.



The resulting system determinant is

$$\Delta = \begin{vmatrix} (r_{c1}+r_{c2}+r_{e2}-r_{m2}) & -(r_{c2}+r_{m1}) & -(r_{e2}-r_{m2}) & 0 \\ -(r_{c2}-r_{m2}) & (r_{c2}+r_{e1}+R_D+r_{b2}) & -(r_{b2}+r_{m2}) & -(R_D) \\ -(r_{e2}) & -(r_{b2}) & (r_{e2}+r_{b2}+R_2) & -(R_2) \\ 0 & -(R_D) & -(R_2) & (R_D+R_2) \end{vmatrix}$$

The simplified form shown below is obtained after the following steps: (1) add third and fourth columns to the second; (2) add second column to the first; (3) add third and fourth rows to the second; (4) add first row to second.

$$\Delta = \begin{vmatrix} (r_{c1} - r_{m1}) & -(r_{c2}+r_{m1}+r_{e2}-r_{m2}) & -(r_{e2}-r_{m2}) & 0 \\ (r_{c1}-r_{m1}+r_{e1}) & -(r_{m1}-r_{e1}) & 0 & 0 \\ 0 & (r_{e2}) & (r_{e2}+r_{b2}+R_2) & -(R_2) \\ 0 & 0 & -(R_2) & (R_D+R_2) \end{vmatrix}.$$

Expanding by the fourth row the system determinant is

$$\Delta = (R_2)^2\Big[(r_{c1}-r_{m1})(r_{m1}-\underline{r_{e1}}) - (r_{c2}+r_{m1}+\underline{r_{e2}}-r_{m2})(r_{c1}-r_{m1}+\underline{r_{e1}})\Big]$$
$$+(R_D+R_2)\Big[(r_{c2}+r_{m1}+\underline{r_{e2}}-r_{m2})(r_{c1}-r_{m1}+\underline{r_{e1}})(r_{e2}+r_{b2}+R_2)$$
$$- r_{e2}(r_{c1}-r_{m1}+\underline{r_{e1}})(\underline{r_{e2}}-r_{m2}) - (r_{c1}-r_{m1})(r_{m1}-\underline{r_{e1}})(r_{e2}+r_{b2}+R_2)\Big].$$

After neglecting the underlined terms, several algerbaic steps will reduce the above to

$$\Delta \approx (1-\alpha_1) r_{c1} r_{c2} \Big\{(1-\alpha_2)\big[R_2(r_{e2}+r_{b2}) + R_D(r_{e2}+r_{b2}+R_2)\big]$$
$$+ \alpha_2 r_{e2}(R_D+R_2)\Big\}.$$

An expression for i_4 is

$$i_4 = -\frac{\theta_0}{\Delta}\begin{vmatrix} (r_{c1}+r_{c2}+\underline{r_{e2}}-r_{m2}) & -(r_{c2}+r_{m1}) & -(\underline{r_{e2}}-r_{m2}) \\ -(r_{c2}-r_{m2}) & (r_{c2}+\underline{r_{e1}+R_D+r_{b2}}) & -(\underline{r_{b2}}+r_{m2}) \\ -(r_{e2}) & -(r_{b2}) & (r_{e2}+r_{b2}+R_2) \end{vmatrix}.$$

Defining the factor in the determinant as D and neglecting the underlined terms we have

$$D \approx r_{c2}(r_{e2}+r_{b2}+R_D)\big[r_{c1}+r_{c2}(1-\alpha_2)\big] - (r_{e2})(\alpha_2 r_{c2})(r_{c2}+\alpha_1 r_{c1})$$
$$+ \alpha_2 (r_{c2})^2 r_{e2} + \alpha_2 (r_{c2})^2 (1-\alpha_2)(r_{b2}) - \alpha_2 r_{c2} r_{b2}\big[r_{c1}+r_{c2}(1-\alpha_2)\big]$$
$$- (r_{c2})(1-\alpha_2)(r_{c2}+\alpha_1 r_{c1})(r_{e2}+r_{b2}+R_2).$$

This can be reduced to

$$D \approx r_{c2} r_{c1} \left\{ R_2\left[1-\alpha_1(1-\alpha_2)\right] + r_{e2}(1-\alpha_1) + r_{b2}\left[1-\alpha_2-\alpha_1(1-\alpha_2)\right]\right\}.$$

Substituting into the following expression for r

$$r = \frac{-e_0}{\lambda_4} = \frac{\Delta}{D},$$

the final form is

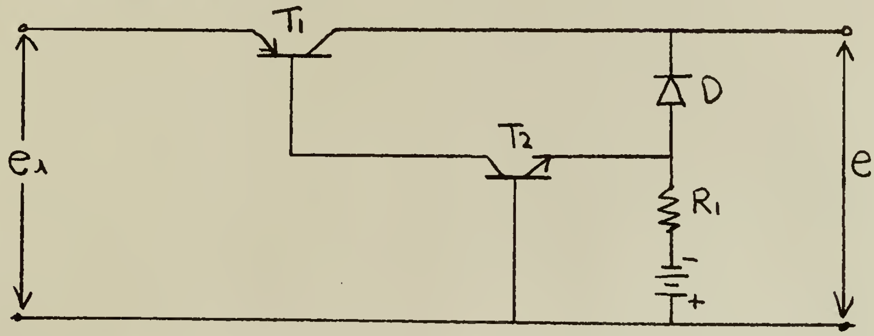
$$r \approx \frac{(1-\alpha_1)\left\{(1-\alpha_2)\left[R_2(r_{e2}+r_{b2}) + R_0(r_{e2}+r_{b2}+R_2)\right] + \alpha_2 r_{e2}(R_0+R_2)\right\}}{R_2\left[1-\alpha_1(1-\alpha_2)\right] + r_{e2}(1-\alpha_1) + r_{b2}\left[1-\alpha_2-\alpha_1(1-\alpha_2)\right]}.$$

Letting R_2 approach infinity will result in the expression for the voltage regulator output impedance factor of the ultimate circuit.

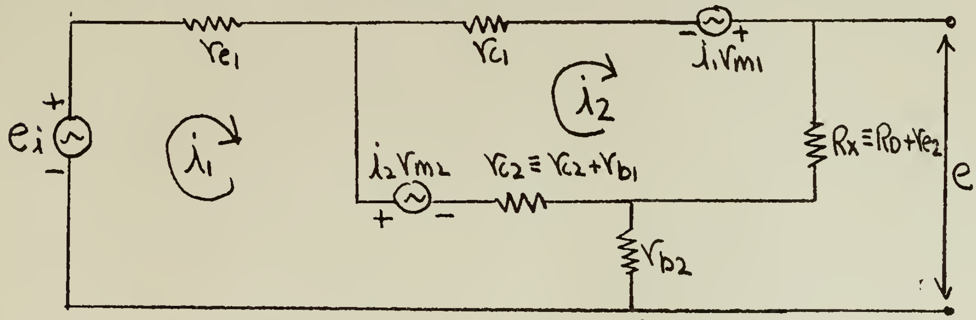
APPENDIX IV

DERIVATIONS FOR COMPLEMENTARY TRANSISTOR SERIES VOLTAGE REGULATOR

The circuit diagram for the complementary transistor series voltage regulator is repeated below.



If we neglect R_1 the following equivalent circuit is applicable for the derivation of the voltage regulator regulation factor.



The system determinant is

$$\Delta = \begin{vmatrix} (\underline{r_{e1}} + r_{c2} + \underline{r_{b2}}) & -(r_{c2} - r_{m2}) \\ -(r_{c2} + r_{m1}) & (r_{c2} - r_{m2} + r_{c1} + \underline{R_x}) \end{vmatrix} .$$

Neglecting those terms underlined Δ may be reduced to

$$\Delta \approx r_{c1} r_{c2} \left[1 - \alpha_1 (1 - \alpha_2) \right] .$$

The expressions for i_1 and i_2 are

$$i_1 = \frac{e_i}{\Delta}\left(r_{c2} - r_{m2} + r_{c1} + R_x\right)$$

$$i_2 = \frac{e_i}{\Delta}\left(r_{c2} + \alpha_1 r_{c1}\right).$$

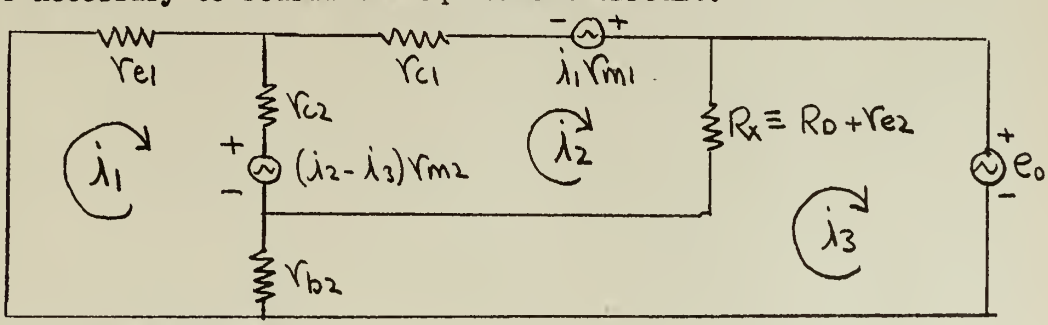
The above relations can be used to solve for $\mathcal{R}$.

$$\mathcal{R} = \frac{e_o}{e_i} = \frac{i_1 r_{b2} + i_2 R_x}{e_i}$$

$$\mathcal{R} = \frac{r_{b2}\left[r_{c2}(1-\alpha_2) + r_{c1} + R_x\right] + R_x\left(r_{c2} + \alpha_1 r_{c1}\right)}{r_{c1} r_{c2}\left[1 - \alpha_1(1-\alpha_2)\right]}$$

$$\mathcal{R} \approx \frac{r_{b2}(1-\alpha_2) + R_o + r_{e2}}{r_{c1}\left[1-\alpha_1(1-\alpha_2)\right]} + \frac{\alpha_1(R_o + r_{e2}) + r_{b2}}{r_{c2}\left[1-\alpha_1(1-\alpha_2)\right]}$$

In order to solve for the voltage regulator output impedance factor it is necessary to redraw the equivalent circuit.



The system determinant is

$$\Delta = \begin{vmatrix} (r_{e1} + r_{c2} + r_{b2}) & -(r_{c2} - r_{m2}) & -(r_{b2} + r_{m2}) \\ -(r_{c2} + r_{m1}) & +(r_{c1} + r_{c2} + R_x - r_{m2}) & -(R_x - r_{m2}) \\ -(r_{b2}) & -(R_x) & +(r_{b2} + R_x) \end{vmatrix}.$$

After adding the second and third columns to the first and then the third row of the resultant determinant to the first, Δ is simplified to

$$\Delta = \begin{vmatrix} (r_{e1}) & -(r_{c2} - r_{m2} + \underline{R_x}) & (\underline{R_x} - r_{m2}) \\ (r_{c1} - r_{m1} + \underline{r_{e1}}) & (r_{c1}) & 0 \\ 0 & -(R_x) & (r_{b2} + R_x) \end{vmatrix}.$$

Neglecting the underlined terms the determinant is expanded into the following form

$$\Delta \approx r_{e1} r_{c1} (r_{b2} + R_x) + \alpha_2 r_{c1} r_{c2} R_x (1-\alpha_1) + r_{c1} r_{c2} (1-\alpha_1)(1-\alpha_2)(r_{b2} + R_x).$$

It is obvious that the first term in the above expression is much smaller than the remaining two. Using this assumption Δ may be manipulated into the form

$$\Delta \approx r_{c1} r_{c2} (1-\alpha_1)\left[R_x + (1-\alpha_2) r_{b2}\right].$$

Now i_3 is equal to

$$i_3 = \frac{-e_0}{\Delta}\left[(\underline{r_{e1}} + r_{c2} + \underline{r_{b2}})(r_{c1} + r_{c2} + \underline{R_x} - r_{m2}) - (r_{c2} - r_{m2})(r_{c2} + r_{m1})\right].$$

If the underlined terms are omitted the above reduces to

$$i_3 = \frac{-e_0 r_{c2} r_{c1}}{\Delta}\left[1 - \alpha_1 (1-\alpha_2)\right].$$

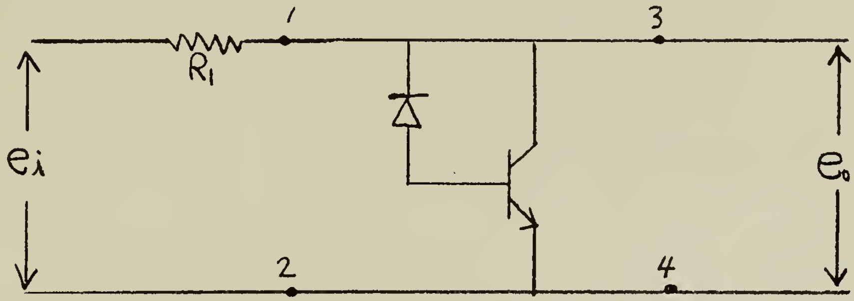
Substituting into the expression for r the final approximate result is obtained.

$$r = \frac{-e_0}{i_3} = \frac{(1-\alpha_1)\left[R_D + r_{e2} + (1-\alpha_2) r_{b2}\right]}{1 - \alpha_1 (1-\alpha_2)}$$

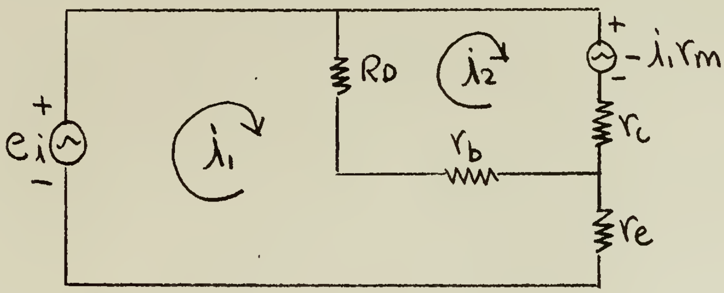
APPENDIX V

DERIVATIONS FOR BASIC TRANSISTOR SHUNT VOLTAGE REGULATOR

The circuit for the basic transistor shunt voltage regulator is shown below.



The effective value of R_D is determined by finding the input impedance at terminals 1 and 2 with 3 and 4 open-circuited. The following is the appropriate equivalent circuit.



The system determinant is

$$\Delta = \begin{vmatrix} (R_D + r_b + r_e) & -(r_b + R_D) \\ -(R_D + r_b + r_m) & (R_D + r_b + r_c) \end{vmatrix}.$$

Solving the determinant results in

$$\Delta = r_c (R_D + r_b + r_e)(1-\alpha) + r_e(\underline{R_D} + \underline{r_b} + r_m)$$

which may be reduced to

$$\Delta \approx r_c \left[r_e + (1-\alpha)(r_b + R_D) \right].$$

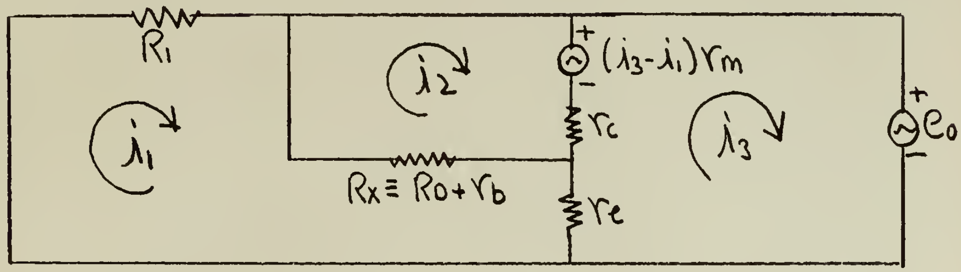
The current i_1 is given by

$$i_1 = \frac{e_i}{\Delta}\left(R_D + r_b + r_c\right) \approx \frac{e_i}{\Delta} r_c \,.$$

Solving for the input impedance which is equal to R_D' we have

$$R_{IN} = \frac{e_i}{i_1} \approx r_e + (1-\alpha)(R_D + r_b) \equiv R_D' \,.$$

The equivalent circuit necessary for determining the value of r is shown below.



The system determinant is

$$\Delta = \begin{vmatrix} (R_1 + R_X + r_e) & -(R_X) & -(r_e) \\ -(R_X + r_m) & +(R_X + r_c) & -(r_c - r_m) \\ -(r_e - r_m) & -(r_c) & +(r_e + r_c - r_m) \end{vmatrix} .$$

Adding the second and third rows to the first gives

$$\Delta = \begin{vmatrix} (R_1) & 0 & 0 \\ -(R_X + r_m) & (R_X + r_c) & -(r_c - r_m) \\ -(r_e - r_m) & -(r_c) & +(r_e + r_c - r_m) \end{vmatrix} .$$

Solving the determinant we can put Δ in the form

$$\Delta = R_1 \left\{ R_X \left[r_e + (1-\alpha) r_c \right] + r_c r_e \right\} .$$

The expression for i_3 is

$$i_3 = \frac{-e_0}{\Delta}\left[(R_1+r_e)(R_x+r_c) + r_c R_x(1-\alpha)\right].$$

It is now possible to solve for r.

$$r = -\frac{e_0}{i_3} = \frac{R_1\left\{R_x\left[r_e + r_c(1-\alpha)\right] + r_c r_e\right\}}{(R_1+r_e)(R_x+r_c) + r_c R_x(1-\alpha)}$$

If both the numerator and denominator are divided by r_c the underlined terms below can be seen to be insignificant. The final result can be recognized as the expected parallel combination of R_1 and R_D'.

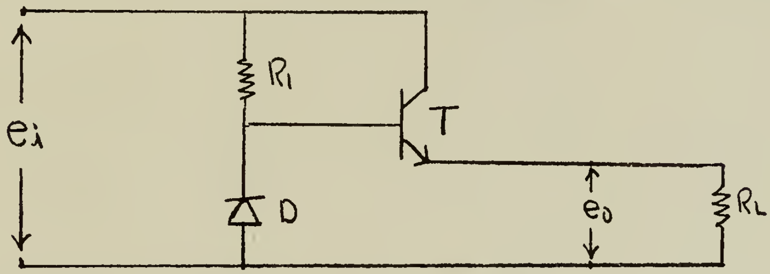
$$r = \frac{R_1\left\{R_x\left[\underline{\frac{r_e}{r_c}} + (1-\alpha)\right] + r_e\right\}}{(R_1+r_e)\left(\underline{\frac{R_x}{r_c}} + 1\right) + R_x(1-\alpha)}$$

$$r \approx \frac{R_1\left[r_e + (1-\alpha)(R_D + r_b)\right]}{R_1 + r_e + (1-\alpha)(R_D+r_b)}$$

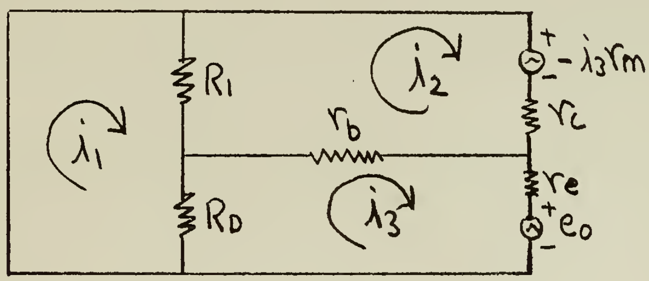
APPENDIX VI

DERIVATIONS FOR THE EMITTER FOLLOWER VOLTAGE REGULATOR

The circuit for the emitter follower regulator is repeated below.



The following equivalent circuit is used to solve for the output impedance factor, r.



The system determinant is

$$\Delta = \begin{vmatrix} (R_1 + R_D) & -(R_1) & -(R_D) \\ -(R_1) & (R_1 + r_c + r_b) & -(r_b + r_m) \\ -(R_D) & -(r_b) & (R_D + r_b + r_e) \end{vmatrix} .$$

Adding the second and third columns to the first we obtain

$$\Delta = \begin{vmatrix} 0 & -(R_1) & -(R_D) \\ (r_c - r_m) & (R_1 + r_c + \underline{r_b}) & -(\underline{r_b} + r_m) \\ (r_e) & -(r_b) & (R_D + r_b + r_e) \end{vmatrix} .$$

If the underlined terms are neglected this may be written as

$$\Delta \approx r_c R_1 \left[(1-\alpha)(R_D + r_b) + r_e \right] + r_c r_b R_D (1-\alpha) + R_D r_e (\underline{R_1} + r_c) .$$

For the regulator to function properly R_1 must be several orders of magnitude smaller than r_c. Omitting underlined portion of the last term permits Δ to be manipulated into the following form.

$$\Delta \approx r_c \left\{ r_e (R_1 + R_D) + (1-\alpha)\left[r_b (R_1 + R_D) + R_1 R_D \right] \right\}$$

The expression for i_3 is

$$i_3 = -\frac{e_0}{\Delta}\left[(R_1 + R_D)(R_1 + r_c + \underline{r_b}) - (R_1)^2 \right].$$

In the following steps we neglect the underlined terms in order to reduce i_3 to a more convenient form.

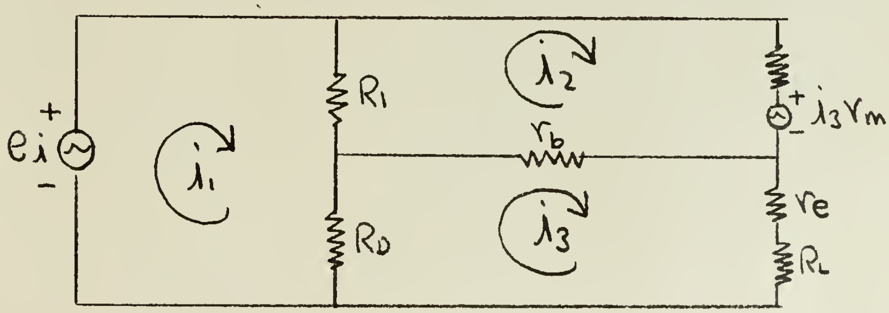
$$i_3 \approx -\frac{e_0}{\Delta}\left[r_c (R_1 + R_D) + R_1 R_D \right] \approx -\frac{e_0}{\Delta} r_c (R_1 + R_D)$$

Substituting into the expression for r and simplifying we have.

$$r = -\frac{e_0}{i_3}$$

$$r \approx r_e + (1-\alpha)\left[r_b + \frac{R_1 R_D}{R_1 + R_D} \right]$$

The load cannot be separated from the regulator in the emitter follower configuration. It is therefore necessary to include R_L and solve for the ratio $\frac{e_o}{e_i}$ rather than follow the usual procedure and finding $\mathcal{R}$. The following is the appropriate equivalent circuit.



The system determinant is

$$\Delta = \begin{vmatrix} (R_1 + R_D) & -(R_1) & -(R_D) \\ -(R_1) & (R_1 + r_c + r_b) & -(r_b + r_m) \\ -(R_D) & -(r_b) & (R_D + r_b + r_e + R_L \end{vmatrix}.$$

The above expression is the same as the Δ used in solving for r except that the term R_L plus r_e appears every place r_e alone did before. The simplified form of the new Δ may be written directly.

$$\Delta \approx r_c \left\{ (1-\alpha)\left[r_b(R_1 + R_D) + (r_e + R_L)(R_1 + R_D) \right] \right\}$$

The current i_3 is equal to

$$i_3 = \frac{e_i}{\Delta}\left[R_1 r_b + R_D(R_1 + r_c + r_b) \right].$$

Neglecting the underlined terms this is closely approximated by

$$i_3 \approx \frac{e_i}{\Delta} R_D r_c .$$

Solving for $\frac{e_o}{e_i}$ we have the final result

$$\frac{e_o}{e_i} \approx \frac{R_L R_D}{(1-\alpha)\left[R_D(R_1 + r_b) + r_b R_1 \right] + (r_e + R_L)(R_1 + R_D)}.$$

This may be rewritten in the form

$$\frac{e_o}{e_i} \approx \frac{R_L}{(1-\alpha)\left[r_b + R_1 + \frac{r_b R_1}{R_D} \right] + (r_e + R_L)\left(\frac{R_1}{R_D} + 1 \right)}.$$

If the approximations

$$R_L \gg r_e$$
$$R_1 \gg R_D$$

are made, it is noted that the first term of the denominator is small with respect to the second. These approximations result in the very simple expression for regulation shown below.

$$\frac{e_o}{e_i} \rightarrow \frac{R_L}{R_L\left(\frac{R_1}{R_D}+1\right)} \rightarrow \frac{R_D}{R_1}$$